DISSERTATIO,
DE
INVENIENDA AREA CURVÆ, QUÆ IN AQUA MOTA MINIMAM PATITUR RESISTENTIAM;

Quam

Conf. Amplisf. Facult. Philof. Reg. Acad. Aboënf.

PRÆSIDE

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publico examini modeste offert

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Smolandia-Svecus.

In Audit. Anatomico die II. Junii MDCCCII,

Horis a. m. folitis.

ABOÆ, typis Frenckellianis.

KONUNGENS
TROMAN, LAGMANNEN och HÅRADSHÖFDINGEN
öfver
Östbo och Wästbo Domsagor,
HÖGÅDLE
Herr ADAM JORDAN KRÖGER,

KONUNGENS
TROTJENARE, RYTTMÅSTAREN
HÖGVÅLBORNE BARON
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samt
KONUNGENS
TROTJENARE, KAPITENEN
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Herr CHRISTER MÖRNER,

Tilegnadt
af
DERAS

ödmjukaste tjenare
CARL FREDRIC EKWURZEL.

In dissertatione nuper edita de invenienda linea curva, quæ in corpore liquido mota minimam patiatur resistentiam, ostensum est, facta abscissa $= x$, ordinata $= y$, & angulo, quem cum linea abscissarum constituit tangens curvæ, $= w$, æquationibus $x = 5,52181 \left(\frac{1}{2} - 2\, Sin\, w^4 + 3\, Sin\, w^2 + \frac{1}{2}\, Cos\, w - \frac{1}{Cos\, w}\right)$ &

$$y = 1 - 5,52181 \left(2\, Sin\, w^3\, Cos\, w - \frac{Sin\, w^3}{2\, Cos\, w^2}\right)$$

illam curvam determinari posse. In hac disquisitione areæ ejus ratio quoque est habita; quare hujus determinationem in natura curvæ perfecte cognoscenda non esse negligendam, inprimis si in construendis navibus adhibetur, putamus. Sequentibus itaque pagellis bona lectoris venia eam computatam præbere constituimus.

Ipsa autem quadratura invenitur integrando elementum areæ ydx, & ponendo post integrationem $x = 2,63006$ & $y = 0$, vel etiam $w = 39°\ 14'\ 36,5''$ (vide dissertationis nominatæ pag. 8). Facta itaque $a = 5,52181$, est

$$x = a \left(\frac{1}{2} - 2\, Sin\, w^4 + 3\, Sin\, w^2 + \frac{1}{2}\, Cos\, w - \frac{1}{Cos\, w}\right),$$

quare, sumtis utrinque fluxionibus secundum vulgares formulas: $d\,(Sin\, w^m) = m\, Sin\, w^{m-1}\, Cos\, w\, dw$, & $d(Cos\, w^m) = -\, m\, Cos\, w^{m-1}\, Sin\, w\, dw$, invenitur

$dx =$

$$dx = a\left(-8\,Sin\,w^2\,Cos\,w + 6\,Cos\,w - \frac{1}{Cos\,w^2} - \tfrac{1}{2}\right) Sin\,w\,dw,$$

unde ob $y = 1 - a\left(2\,Cos\,w - \dfrac{1}{2\,Cos\,w^2}\right) Sin\,w^3$ habetur

$$ydx = 16\,a^2\,Sin\,w^2\,Cos\,w^2\,dw - 4a^2\,Sin\,w^6\,Cos\,w^{-1}\,dw$$
$$-\,12\,a^2\,Sin\,w^4\,Cos\,w^2\,dw + a^2\,Sin\,w^4\,Cos\,w\,dw +$$
$$5\,a^2\,Sin\,w^4\,Cos\,w^{-1}\,dw - \tfrac{1}{4}\,a^2\,Sin\,w^4\,Cos\,w^{-2}\,dw -$$
$$-\,\tfrac{1}{2}\,a^2\,Sin\,w^4\,Cos\,w^{-4}\,dw - 8\,a\,Sin\,w^3\,Cos\,w\,dw +$$
$$6\,a\,Sin\,w\,Cos\,w\,dw - \tfrac{1}{2}\,a\,Sin\,w\,dw - a\,Sin\,w\,Cos\,w^{-2}\,dw.$$

Integratis quibuscunque hujus æquationis terminis obtinetur ipsa area. Est autem

$$\int Sin\,w^6\,Cos\,w^2\,dw = -\tfrac{1}{8}\,Sin\,w^5\,Cos\,w^3 + \tfrac{5}{8}\int Sin\,w^4\,Cos\,w^2\,dw;$$
$$\int Sin\,w^4\,Cos\,w^2\,dw = -\tfrac{1}{6}\,Sin\,w^3\,Cos\,w^3 + \tfrac{1}{2}\int Sin\,w^2\,Cos\,w^2\,dw;$$
$$\int Sin\,w^2\,Cos\,w^2\,dw = -\tfrac{1}{4}\,Sin\,w\,Cos\,w^3 + \tfrac{1}{4}\int Cos\,w^2\,dw;$$
$$\int Cos\,w^2\,dw = \tfrac{1}{2}\,Sin\,w\,Cos\,w + \tfrac{1}{2}\,w;$$ adeoque

$$16\,a^2\int Sin\,w^6\,Cos\,w^2\,dw = -2a^2\,Sin\,w^5\,Cos\,w^3 - \tfrac{5}{3}\,a^2\,Sin\,w^3\,Cos\,w^3$$
$$-\,\tfrac{5}{4}\,a^2\,Sin\,w\,Cos\,w^3 + \tfrac{5}{8}\,a^2\,Sin\,w\,Cos\,w + \tfrac{5}{8}\,a^2\,w.$$

Similiter est

$$\int Sin\,w^6\,Cos\,w^{-1}\,dw = -\tfrac{1}{5}\,Sin\,w^5 + \int Sin\,w^4\,Cos\,w^{-1}\,dw;$$
$$\int Sin\,w^4\,Cos\,w^{-1}\,dw = -\tfrac{1}{3}\,Sin\,w^3 + \int Sin\,w^2\,Cos\,w^{-1}\,dw;$$
$$\int Sin\,w^2\,Cos\,w^{-1}\,dw = -Sin\,w + \int Cos\,w^{-1}\,dw = -Sin\,w +$$
$$Log\,hyp.\,Tg\,(45^\circ + \tfrac{1}{2}\,w);$$ adeoque

$$-\,4\,a^2\int Sin\,w^6\,Cos\,w^{-1}\,dw = \tfrac{4}{5}\,a^2\,Sin\,w^5 + \tfrac{4}{3}\,a^2\,Sin\,w^3$$

+

$+4a^2 \, Sin \, w - 4a^2 \, Log. \, hyp. \, Tg \, (45^\circ + \frac{1}{2} w)$. Ulterius invenitur $-12a^2 \int Sin \, w^4 \, Cos \, w^2 \, dw = 2a^2 \, Sin \, w^3 \, Cos \, w^3 + \frac{3}{2} a^2 \, Sin \, w \, Cos \, w^3 - \frac{3}{4} a^2 \, Sin \, w - \frac{3}{4} a^2 w$. Est quoque $a^2 \int Sin \, w^4 \, Cos \, w \, dw = \frac{1}{5} a^2 \, Sin \, w^5$. Præterea habetur $5a^2 \int Sin \, w^3 \, Cos \, w^{-1} dw = -\frac{5}{3} a^2 \, Sin \, w^3 - 5a^2 \, Sin \, w + 5a^2 \, Log. \, hyp \, Tg \, (45^\circ + \frac{1}{2} w)$; nec non

$\int Sin \, w^4 \, Cos \, w^{-2} dw = Sin \, w^3 \, Cos \, w^{-1} - 3 \int Sin \, w^2 \, dw$;

$\int Sin \, w^2 \, dw = -\frac{1}{2} Sin \, w \, Cos \, w + \frac{1}{2} w$, &

$-\frac{1}{4} a^2 \int Sin \, w^4 \, Cos \, w^{-2} dw = -\frac{1}{4} a^2 \, Sin \, w^3 \, Cos \, w^{-1} - \frac{3}{8} a^2 \, Sin \, w \, Cos \, w + \frac{3}{8} a^2 w$. Deinde invenitur

$\int Sin \, w^4 \, Cos \, w^{-4} dw = \frac{1}{3} Sin \, w^3 \, Cos \, w^{-3} - \int Sin \, w^2 \, Cos \, w^{-2} dw$;

$\int Sin \, w^2 \, Cos \, w^{-2} dw = Sin \, w \, Cos \, w^{-1} - w$; adeoque

$-\frac{1}{2} a^2 \int Sin \, w^4 \, Cos \, w^{-4} dw = -\frac{1}{6} a^2 \, Sin \, w^3 \, Cos \, w^{-3} + \frac{1}{2} a^2 \, Sin \, w \, Cos \, w^{-1} - \frac{1}{2} a^2 w$, ut etiam

$\int Sin \, w^3 \, Cos \, w \, dw = \frac{1}{4} Sin \, w^4$; atque

$-8a \int Sin \, w^3 \, Cos \, w \, dw = -2a \, Sin \, w^4$;

$6a \int Sin \, w \, Cos \, w \, dw = -3a \, Cos \, w^2$;

$-\frac{1}{2} a \int Sin \, w \, dw = \frac{1}{2} a \, Cos \, w$. Est tandem

$\int Sin \, w \, Cos \, w^{-2} dw = Cos \, w^{-1}$, atque

$-a \int Sin \, w \, Cos \, w^{-2} dw = -a \, Cos \, w^{-1}$ (*).

 Si

(*) Cfr. Specimen *de integratione fluxionum formæ*

Si in summam jam colliguntur omnia hæc integralia, invenitur area quæsita:

$$\int y\,dx = -2a^2 \,Sin\, w^5 \,Cos\, w^3 - \tfrac{5}{3} a^2 \,Sin\, w^3 \,Cos\, w^3$$
$$- \tfrac{5}{4} a^2 \,Sin\, w \,Cos\, w^3 + \tfrac{5}{8} a^2 \,Sin\, w \,Cos\, w + \tfrac{5}{8} a^2 w$$
$$+ \tfrac{4}{5} a^2 \,Sin\, w^5 + \tfrac{4}{3} a^2 \,Sin\, w^3 + 4 a^2 \,Sin\, w$$
$$- 4 a^2 \,Log.\, hyp.\, Tg\,(45^\circ + \tfrac{1}{2} w) + 2 a^2 \,Sin\, w^3 \,Cos\, w^3$$
$$+ \tfrac{3}{2} a^2 \,Sin\, w \,Cos\, w^3 - \tfrac{3}{4} a^2 \,Sin\, w \,Cos\, w - \tfrac{3}{4} a^2 w$$
$$+ \tfrac{1}{5} a^2 \,Sin\, w^5 - \tfrac{5}{3} a^2 \,Sin\, w^3 - 5 a^2 \,Sin\, w$$
$$+ 5 a^2 \,Log.\, hyp.\, Tg\,(45^\circ + \tfrac{1}{2} w) - \tfrac{1}{4} a^2 \,Sin\, w^3 \,Cos\, w^{-1}$$
$$- \tfrac{3}{8} a^2 \,Sin\, w \,Cos\, w + \tfrac{3}{8} a^2 w - \tfrac{1}{6} a^2 \,Sin\, w^3 \,Cos\, w^{-3}$$
$$+ \tfrac{1}{2} a^2 \,Sin\, w \,Cos\, w^{-1} - \tfrac{1}{2} a^2 w - 2a \,Sin\, w^4 - 3a \,Cos\, w^2$$
$$+ \tfrac{1}{2} a \,Cos\, w - a \,Cos\, w^{-1} = -2 a^2 \,Sin\, w^5 \,Cos\, w^3$$
$$+ \tfrac{1}{3} a^2 \,Sin\, w^3 \,Cos\, w^3 - \tfrac{1}{4} a^2 \,Sin\, w^3 \,Cos\, w^{-1}$$
$$- \tfrac{1}{6} a^2 \,Sin\, w^3 \,Cos\, w^{-3} + \tfrac{1}{4} a^2 \,Sin\, w \,Cos\, w^3 - \tfrac{1}{2} a^2 \,Sin\, w \,Cos\, w$$
$$+ \tfrac{1}{2} a^2 \,Sin\, w \,Cos\, w^{-1} + a^2 \,Sin\, w^5 - 2 a \,Sin\, w^4$$
$$- \tfrac{1}{3} a^2 \,Sin\, w^3 - a^2 \,Sin\, w - 3 a \,Cos\, w^2 + \tfrac{1}{2} a \,Cos\, w$$
$$- a \,Cos\, w^{-1} - \tfrac{1}{4} a^2 w + a^2 \,Log.\, hyp.\, Tg\,(45^\circ + \tfrac{1}{2} w) + A,$$

denotante A quantitatem constantem integralium corrigendorum causa additam. Ut determinetur A, monemus aream curvæ evanescere debere facta abscissa $x = 0$, hoc est, posito angulo $w = 0$, quando habemus pro Sinu toto $= 1$, $Sin\, w = 0$, $Cos\, w = 1$, $Log.\, hyp.\, Tg\,(45^\circ + \tfrac{1}{2} w) = 0$; adeo-

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$(Sin\, Z)^m . (Cos\, Z)^n\, dZ$, *Præside* M J. WALLENIO & *Respondente* J. H. LINDQVIST, *Aboæ* 1768 editum.

adeoque his institutis substitutionibus: $o = -\frac{7}{2}a + A$, unde invenitur $A = \frac{7}{2}a$. Correcta area erit itaque:

$$\int y\,dx = -2a^2 \, Sin\,w^5 \, Cos\,w^3 + \tfrac{1}{3}a^2 \, Sin\,w^3 \, Cos\,w^3$$
$$- \tfrac{1}{4}a^2 \, Sin\,w^3 \, Cos\,w^{-1} - \tfrac{1}{6}a^2 \, Sin\,w^3 \, Cos\,w^{-3}$$
$$+ \tfrac{1}{4}a^2 \, Sin\,w \, Cos\,w^3 - \tfrac{1}{2}a^2 \, Sin\,w \, Cos\,w$$
$$+ \tfrac{1}{2}a^2 \, Sin\,w \, Cos\,w^{-1} + a^2 \, Sin\,w^5 - 2a \, Sin\,w^4$$
$$- \tfrac{1}{3}a^2 \, Sin\,w^3 - a^2 \, Sin\,w - 3a \, Cos\,w^2 + \tfrac{1}{7}a \, Cos\,w$$
$$- a \, Cos\,w^{-1} - \tfrac{1}{4}a^2 w + a^2 \, Log.hyp.\, Tg\,(45^\circ + \tfrac{1}{2}w) + \tfrac{7}{2}a.$$

Ut jam completa habeatur area, ponatur $w = 39^\circ\, 14'\, 36{,}5''$, quo fit $Log.\,Sin\,w = 0{,}8011409 - 1$, atque $Log.\,Cos\,w = 0{,}8890016 - 1$. Quum etiam sit $Log.\,a = 0{,}7420815$; invenitur

$L2 = 0{,}3010300$	$L\frac{1}{3} = 0{,}5228787 - 1$
$La^2 = 1{,}4841630$	$La^2 = 1{,}4841630$
$L\,Sin\,w^5 = 0{,}0057045 - 1$	$L\,Sin\,w^3 = 0{,}4034227 - 1$
$L\,Cos\,w^3 = 0{,}6670048 - 1$	$L\,Cos\,w^3 = 0{,}6670048 - 1$
$0{,}4579023$	$0{,}0774692$
$2a^2\,Sin\,w^5\,Cos\,w^3 = 2{,}870135.$	$\frac{1}{3}a^2\,Sin\,w^3\,Cos\,w^3 = 1{,}195279.$
$L\frac{1}{4} = 0{,}3979400 - 1$	$L\frac{1}{4} = 0{,}3979400 - 1$
$La^2 = 1{,}4841630$	$La^2 = 1{,}4841630$
$L\,Sin\,w^3 = 0{,}4034227 - 1$	$L\,Sin\,w = 0{,}8011409 - 1$
$L\,Cos\,w^{-1} = 0{,}1109984$	$L\,Cos\,w^3 = 0{,}6670048 - 1$
$0{,}3965241$	$0{,}3502487$
$\frac{1}{4}a^2\,Sin\,w^3\,Cos\,w^{-1} = 2{,}491863$	$\frac{1}{4}a^2\,Sin\,w\,Cos\,w^3 = 2{,}240003.$

 $L\frac{1}{6}$

$L\,\tfrac{1}{6} = 0{,}2218487 - 1$
$L\,a^2 = 1{,}4841630$
$L\,Sin\,w^3 = 0{,}4034227 - 1$
$L\,Cos\,w^{-3} = 0{,}3329952$
$\overline{\qquad 0{,}4424296}$

$\tfrac{1}{6}a^2\,sin\,w^3\,cos\,w^{-3} = 2{,}769680$

$L\,\tfrac{1}{2} = 0{,}6989700 - 1$
$L\,a^2 = 1{,}4841630$
$L\,Sin\,w = 0{,}8011409 - 1$
$L\,Cos\,w^{-1} = 0{,}1109984$
$\overline{\qquad 1{,}0952723}$

$\tfrac{1}{2}a^2\,sin\,w\,Cos\,w^{-1} = 12{,}452960.$

$L\,\tfrac{1}{2} = 0{,}6989700 - 1$
$L\,a^2 = 1{,}4841630$
$L\,Sin\,w = 0{,}8011409 - 1$
$L\,Cos\,w = 0{,}8890016 - 1$
$\overline{\qquad 0{,}8732755}$

$\tfrac{1}{2}a^2\;Sin\,w\;Cos\,w = 7{,}469223.$

$L\,a^2 = 1{,}4841630$
$L\,Sin\,w^5 = 0{,}0057045 - 1$
$\overline{\qquad 0{,}4898675}$

$a^2\;Sin\,w^5 = 3{,}089353.$

$L\,2 = 0{,}3010300$
$L\,a = 0{,}7420815$
$L\,Sin\,w^4 = 0{,}2045636 - 1$
$\overline{\qquad 0{,}2476751}$

$2\,a\,Sin\,w^4 = 1{,}768785.$

$L\,\tfrac{1}{2} = 0{,}6989700 - 1$
$L\,a = 0{,}7420815$
$L\,Cos\,w = 0{,}8890015 - 1$
$\overline{\qquad 0{,}3300531}$

$\tfrac{1}{2}\,a\;Cos\,w = 2{,}138223.$

$L\,\tfrac{1}{3} = 0{,}5228787 - 1$
$L\,a^2 = 1{,}4841630$
$L\,Sin\,w^3 = 0{,}4034227 - 1$
$\overline{\qquad 0{,}4104644}$

$\tfrac{1}{3}\,a^2\;Sin\,w^3 = 2{,}573146.$

Facto *Log hyp.* $Tg\,(45^\circ + \tfrac{1}{2}w) = 0{,}7457675 = c$, eft

$L\,a^2 = 1{,}4841630$
$L\,c = 0{,}8726035 - 1$
$\overline{\qquad 1{,}3567665}$

$a^2\;c = 22{,}738742.$
$\tfrac{7}{2}\;a = 19{,}326335.$

La =

$$
\begin{aligned}
L\,a^2 &= 1{,}4841630\\
L\ Sin\ w &= 0{,}8011409-1\\
\hline
&\ 1{,}2853039
\end{aligned}
$$

$$a^2\ Sin\ w = 19{,}288740.$$

$$
\begin{aligned}
L\ 3 &= 0{,}4771213\\
L\ a &= 0{,}7420815\\
L\ Cos\ w^2 &= 0{,}7780032-1\\
\hline
&\ 0{,}9972060
\end{aligned}
$$

$$3\,a\ Cos\,w^2 = 9{,}935875.$$

$$
\begin{aligned}
L\,a &= 0{,}7420815\\
L\ Cos\,w^{-1} &= 0{,}1109984\\
\hline
&\ 0{,}8530799
\end{aligned}
$$

$$a\ Cos\,w^{-1} = 7{,}129843.$$

$$
\begin{aligned}
L\ \tfrac{1}{4} &= 0{,}3979400-1\\
L\ a^2 &= 1{,}4841630\\
L\ w &= 0{,}8356447-1\\
\hline
&\ 0{,}7177477
\end{aligned}
$$

$$\tfrac{1}{4}\,a^2\ w = 5{,}220927$$

In ſummam itaque collectis omnibus quantitatibus poſitivis & negativis, inveniuntur

&

$$
\begin{array}{r}
+\ 63,180895 \\
-\ 61,518217 \\
\hline
\end{array}
$$

& ipſa area = 1,662678, ſumto pro unitate quadrato diſtantiæ inter lineam abſciſſarum & illud punctum curvæ, ubi eſt $w = 0$.