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Tax Policy Design in a Hierarchical Model with Occupational Decisions *

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Abstract

This study incorporates occupational decisions in a hierarchical model to investigate distortions in the tax policy design. The economy has two sectors, wage-earners and self-employment, where evasion is only possible in the latter. The optimal audit split self-employed between audited and non-audited distorted the occupational decision in the last group. The marginal tax rate is smaller than the case without occupational choices, showing that not considering it produces an upward tax bias. The optimal IRS budget does not allow for auditing the entire self-employment sector but is larger than the case without occupational choices. Production efficiency is not attained at the optimum because occupational decisions are distorted. This result contradicts the Diamond-Mirrlees theorem. Finally, differential taxation is a Pareto improvement but implies higher taxes for self-employment. This result increases distortions in the optimal allocation of agents compared to the former setting.

Keywords: Tax Evasion, Hierarchical Model, Tax Policy, Auditing, Income Taxation, Occupational Choice, Production efficiency.
JEL Classification: H26, H21, H83.

1 Introduction

Increasing tax revenue has been an important issue for governments because of increasing social expenditures, paying debts, and saving for future spending. Within this scope, reducing tax evasion or tax avoidance is central to state policies. For instance, after the economic crisis of 2008, developed countries focused on fighting tax evasion and understanding its mechanisms to increase tax revenues

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(Slemrod, 2019). This is mainly because the fight against tax evasion increases tax revenue through effectiveness instead of changing tax policy, which, bureaucratically, takes more time. Therefore, a central scope in tax policy is, broadly speaking, finding ways for compliance policy to be more efficient.

However, different occupations do not provide the same tax evasion opportunities. It is well established that occupations with income self-reporting are prone to evasion since agents can strategically declare income to pay fewer taxes. Kleven et al. (2011) and Slemrod (2007) provide evidence that evasion in third-party income reports is low, and the evasion problem arises mainly in occupations with income self-reporting.[1] Therefore, designing an optimal tax administration policy to diminish tax evasion must consider different opportunities for evasion across occupations and include some consideration of those in the analysis. However, the literature has not placed enough importance on inquiring about both issues in the same context, producing a lack of information about distortions because of coexisting evasion and different occupations.

The current study tries to close this gap by incorporating occupational decisions (self-employment vs. wage-earner) and tax evasion into a tax policy design model. The starting point is a hierarchical model formerly developed by Sanchez and Sobel (1993) (S-S henceforth), in which a government (the principal) interacts with a tax agency (a mediator), and this agency with taxpayers (agents). Hence, we incorporate in S-S´s model an endogenous occupational decision to capture differences in opportunities for evasion across sectors. The main interest is to investigate possible distortions in tax policy: audits, the marginal tax rate, and the budget for tax authority (henceforth the IRS), as well as its consequences for occupational decisions. Hierarchical models help in performing normative analyses (Melumad and Mookherjee, 1989), therefore this paper focuses on the normative implications of including evasion and occupational choices in tax policy design. Simultaneously, it intends to investigate other relevant issues, namely the forces behind the optimal policy in this setting, possible distortions in the optimal allocation of agents driven by the institutional background, and the mechanisms behind these distortions.

Since the hierarchical model developed in S-S is used as a benchmark, it is essential to describe their principal results early in this document to understand better the implication of including occupation decisions. In this model, the IRS maximizes tax revenue and interacts with a government in an economy with only one sector and a continuum of risk-neutral taxpayers. Below a threshold, optimal audits take the maximum value, and after that agents do not face audits. This kind of form is a so-called cutoff audit function. In addition, the budget for the IRS is insufficient to audit all self-employed agents. Finally, they find that the optimal marginal rate is equal to one, and all the redistribution is through the public goods provision and a subsidy that is equal for all agents.

Incorporating occupational decision in a tax policy design model captures the idea that different sectors have different evasion opportunities. In this sense, analysing those distortions helps to understand some empirical facts and improve the effectiveness of the tax compliance policy. This study shows that the coexistence of occupational decisions and evasion distorts optimal tax policy.

[1] Regarding strategic declarations, Kleven (2016) provides a review on bunching, Bastani and Waldenström (2021) show that bunching behavior is connected with agents' ability, and Almunia and Lopez-Rodriguez (2018) show that firms make strategic income declarations to avoid more vigorous audits. Those papers provide evidence that strategic income declaration exists and depends on the agents' ability to manipulate it.

First, not considering occupational decisions produces an upward bias on taxes. Second, the budget for the IRS is larger than if occupational choices are not considered, but not enough to audit all self-employed. Both elements are evidence that a non-optimal policy plays in favor of evasion and distorts agents towards self-employment. This comes from the audit distortions: the impossibility of auditing all self-employment results in the high-income self-employed paying the same taxes, increasing the utility and distorting occupational decisions in this zone. Differential taxation is a Pareto improvement in this setting but implies higher taxes in the self-employment sector, distorting the allocation of workers. Finally, at the optimum, the optimal allocation of workers is slanted in favor of self-employment, and production efficiency is not attained in the second-best tax optimum. This result contradicts the Diamond-Mirrlees theorem (Diamond and Mirrlees, 1971a,b).

This paper solves a three-stage game in which the government, the IRS, and a continuum of risk-neutral taxpayers interact. Taxpayers choose between being self-employed or wage-earners, where evasion is only possible in the self-employment sector. In the first stage, the government maximizes a social welfare function and commits to a marginal tax rate, a level of public goods, and the budget for the IRS. In the second stage, the IRS determines an audit function to maximize the expected tax collection subject to its budget constraint. Finally, in the third stage, taxpayers maximize their utility by choosing an occupation. If they are self-employed, they also choose their income declaration.

The IRS finds a direct incentive-compatible mechanism composed of a direct audit function and effective taxes, which are the agent's tax liability. Since productivity is private information, it is necessary to use the revelation principle and find a mechanism composed of audits and effective taxes that incentivizes agents to reveal their income. As in S-S's work, if the IRS has an insufficient budget, the audit function takes its maximum level until a threshold wage, and after this it is equal to zero. This form results in the effective tax being the same for taxpayers who do not face audits, generating distortion in occupational decisions for (potential) high-income self-employed in favor of self-employment. This result holds in two extensions: a) if the audit cost is monotonically non-decreasing with self-employment income, and b) if the fine rate is increasing in the self-employment wage, it must be bounded from above.

The optimal public good provision is the same as in the first-best because agents are risk-neutral. This result comes from the non-alteration of the marginal rate of substitution between public goods and private consumption (Boadway and Keen, 1993). In this context, the result reinforces that tax evasion distorts income distribution because wage-earners and audited self-employed increase their burden to finance public goods.

The optimal marginal tax rate is smaller than one and can be characterized by two effects: welfare and revenue. Taxes are biased upward if the government does not consider occupational decisions. This conclusion comes from the comparison with S-S's model. The difference emerges because higher taxes raise tax revenue but also downward welfare and increase the incentive to evade. Hence, welfare and revenue effects due to occupational decisions produce lower taxes. Differential taxation, i.e., one marginal tax rate for each occupation, is a Pareto improvement, but it implies a higher marginal tax rate in the self-employment sector. A rise in self-employment taxes induces taxpayers to move to the wage-earner sector, reducing evasion incentives. This result produces an even more significant distortion in the allocation of agents than the former scenario.

The optimal IRS budget is insufficient to audit the entire self-employed sector, and three effects characterize its level: behavioral, mechanical, and welfare. Even though the budget cannot allow for auditing all self-employed, this level is higher than the case without occupational decisions. This fact comes from the marginal gains for an extra dollar/euro to the IRS. When audit increases, more taxpayers decide their occupation efficiently, paying higher taxes and increasing the effective tax for those not audited. However, this effect is decreasing in the extra budget, so it is not optimal to audit the entire self-employment sector.

Distortions through audits contradict the Diamond-Mirrlees theorem because it is impossible to attain production efficiency in a second-best tax scheme. At the equilibrium, the IRS is underbudgeted, and (potential) high-income self-employed have incentives to be self-employed. This result is not solved by extending the model to differential taxation, creating more distortions under this scheme. Hence, because of the institutional design, high-skilled wage-earner workers move to less productive occupations. The critical element for this result is the non-optimality of giving enough resources to the IRS.

This study brings together two strands of literature. First, this paper expands the theoretical optimal tax administration literature by demonstrating and explaining distortions due to coexisting evasion and occupational decisions. In this area, all papers build on the classical contributions of Allingham and Sandmo (1972) and Srinivasan (1973), and the extension of Yitzhaki (1974). Most papers focus on the evasion effect on self-employment, modeling only one sector and demonstrating the consequences for auditing and taxation.[2] Mainly, they demonstrate that audits have to be strongest in sectors that are more prone to finding evaders, and the tax system becomes regressive because of evasion. Along this line, like this paper, Chander and Wilde (1998) provides a general characterization of tax administration policy without a hierarchical model and occupational decision. The closest work to ours is Sanchez and Sobel (1993), which is the natural benchmark since we extend their model by incorporating occupational decisions. This paper expands this literature by showing that occupational decisions produce smaller taxes and a larger budget for the IRS, and differential taxation is a Pareto improvement.

The second strand of literature relies on the evasion effect on the labor market, particularly on the extensive margin decision, in a setting with tax administration policy. This line began with the seminal contribution of Sandmo (1981), later followed by theoretical developments by Boadway et al. (1991); Cowell (1985) and Pestieau and Possen (1991), Parker (1999), who includes a simulation for the UK, and Watson (1985) and Kesselman (1989) in a general equilibrium framework. Recently, Casamatta (2021) incorporated occupational decisions in an optimal income taxation model similar to that of Chander and Wilde (1998), but without an IRS. Therefore, this paper contributes by studying distortions in occupational decisions in a hierarchical model. In this sense, the underbudgeted IRS result is critical because it raises the incentives to become self-employed. Moreover, and in a similar vein as Gomes et al. (2017) and Best et al. (2015), it also is demonstrated that those distortions affect production efficiency in a second-best optimal taxation setting, contradicting the

[2]Some examples are Border and Sobel (1987); Cremer et al. (1990); Landsberger et al. (2000); Macho-Stadler and Perez-Castrillo (1997); Mookherjee and Png (1990) and Reinganum and Wilde (1985, 1986).

Diamond-Mirrlees theorem.

The rest of the paper is organized as follows. Section 2 presents the model and provides the first-best as a benchmark. Section 3 solves the IRS problem, showing the characterization of the optimal audit policy and the effective tax. Section 4 solves the government's problem, obtaining the optimal marginal tax rate, the IRS budget, and the optimal provision of public goods. Section 5 explains the consequences of the equilibrium and connects it with some empirical results. Section 6 explains the optimality of differential taxation, and the requirements for the audit cost and fine rate to obtain the same audit solution. Finally, section 7 concludes.

2 Model

The economy has three agents: the government, the IRS, and a continuum of risk-neutral taxpayers of mass one, which interact in a hierarchical tax administration model. Unlike inSanchez and Sobel (1993), two sectors exist: self-employment and wage-earner, indexed by $i \in \{s, w\}$, where taxpayers decide where to work. Since the IRS knows perfectly in which sector each taxpayer works, and in sector w employers report taxpayers' income, it is impossible to evade taxes in sector w.[3] However, tax evasion is possible in sector s, and income verification is costly. It is assumed that the government has no resources (technology, knowledge, and expertise, among other things) to perform the audit process efficiently. Therefore, the existence of a specialized agency to enforce tax compliance is necessary.

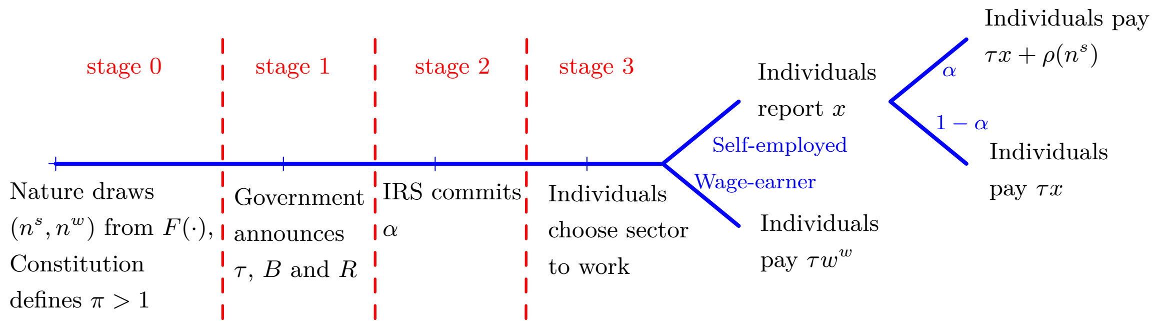
An interesting way to analyse this setting is through differential taxation, i.e., one tax scheme for each occupation. This approach could encompass each occupation's inherent differences, for instance, evasion. Along this line, Gomes et al. (2017) find the optimal differential taxation in an occupational decisions model similar to the one in this paper but without evasion. Appendix A uses Gomez and coauthors' paper as a benchmark to demonstrate that introducing evasion in this setting is worth it. Indeed, evasion necessitates a tax compliance policy, which alters optimal taxes and increases government costs.

The timing of the model is explained now to simplify the rest of the description. In the first stage, the government chooses a marginal tax rate, τ, a budget for the IRS, B, and a level of public goods provision, R. The government cannot impose a marginal tax higher than one.[4] The IRS selects an audit function in the second stage, α. Finally, in the third stage, taxpayers decide in which sector to work, and if self-employed, also choose their income declaration. Finally, taxpayers pay their taxes. Wage-earner and non-audited self-employed only pay taxes on their declared income and audited self-employed also pay penalties. Figure 1 shows this game.

[3] It is assumed that collusion between firms and workers to underreport wages and divide the evaded amount is impossible.

[4] Mirrlees (1971) obtains this as a result of an optimal non-linear tax schedule. This assumption is not restrictive. Afterward, it will be shown to hold at the optimum.

Figure 1: Timing of the Game



2.1 Agents

Nature randomly chooses a pair of productivities $n = (n^s, n^w)$ for each taxpayer. Each term represents productivity in the self-employment and wage-earner sectors, respectively, and is drawn independently from a distribution function. Formally, n^i comes from a cumulative distribution function F^i, twice continuously differentiable (positive derivative), with support $[\underline{n}^i, \overline{n}^i]$ and $\underline{n}^i \geq 0$ for $i \in \{s, w\}$. We assume the existence of common support or $\underline{n}^i < \underline{n}^j$ with $i \neq j$ and $i, j \in \{s, w\}$ for a mass of taxpayers larger than zero. This function is common knowledge in the model.

Taxpayers consume a homogeneous good whose production is linear in labor, and its price is normalized to one. Also, it is assumed that this market is perfectly competitive. Hence, the wage in each sector is equal to one, and the agent's income (w^i) is the same as the agent's productivity (hereafter, we refer to wage instead of agent's income).

Taxpayers decide their occupation by comparing both sectors and choosing which gives them the highest utility. Each taxpayer offers an inelastic labor supply equal to one in each sector.[5] Since taxpayers are risk-neutral, their utility can be written as a quasi-linear function in consumption, $C : [\underline{w}^i, \overline{w}^i] \to \mathbb{R}_+$, with $i \in \{s, w\}$. Let $\phi \in \mathbb{R}_+$ be the benefit for a certain level of public goods $R \in \mathbb{R}_+$, which is increasing and concave (i.e., $\phi' > 0$ and $\phi'' < 0$). From these, the agents' utility is $U(w, R) = C(w) + \phi(R)$.

In the wage-earner sector, consumption is equal to the after-tax income, $C(w^w) = w^w(1 - \tau)$. Consequently, the utility in this sector is

$$U^w(w^w, R) = w^w(1 - \tau) + \phi(R)$$

Note that, since wage-earners do not make any decision, $U^w(w^w, R)$ is their indirect utility. In the self-employment sector, taxpayers make an income declaration $x : [\underline{n}^s, \overline{n}^s] \to \mathbb{R}_+$. The IRS does not reward over-reporting, implying that any declaration above real productivity is a dominated strategy, hence $x \leq w^s$. Self-employed face an audit with probability $\alpha \in [0, 1]$, and if the IRS audits them, it immediately discovers whether they have evaded. Audited self-employed pay a penalty $\rho \in \mathbb{R}$, composed by a fine rate $\pi > 1$ over the evaded taxes. The penalty takes the

[5] Implicitly, it has been taken as fact that effective wage is equal to productivity and we write w or n indifferently.

following form

$$\rho(w^s) = \begin{cases} \pi\tau\left(w^s - x\right) & \text{if } x < w^s \\ 0 & \text{if } x \geq w^s \end{cases}$$

The penalty function establishes that, if a taxpayer declares their true income, the penalty is zero. Otherwise, the penalty is a proportion π of the evaded taxes $\tau\left(w^s - x\right)$.

Let us define $\bar{U}^s(w^s, R)$ as the utility with the optimal income declaration in sector s. Formally, this utility level is given by the following

$$\bar{U}^s(w^s, R) = \max_{x \leq w^s} \left[w^s - \tau x - \varphi(x)\max\left\{\pi\tau\left(w^s - x\right), 0\right\} + \phi(R)\right]$$

where $x(w^s)$ is the solution to this problem. In this case, $C(w^s) = w^s - \tau x - \varphi(x)\max\left\{\pi\tau\left(w^s - x\right), 0\right\}$.

2.2 IRS

The IRS chooses the audit function to maximize the expected tax collection. Define the audit function as $\alpha : [\underline{w}^s, \overline{w}^s] \to [0, 1]$. The IRS has a budget B to finance the cost of auditing, where the cost of auditing a taxpayer c is linear and constant. If the IRS does not use its entire budget, the excess must be returned to the government along with the tax collection.

Self-employed agents can hide information and optimally choose $x(w^s) < w^s$. Therefore, the IRS needs to use a method that incentivizes the self-employed to declare their true wages. Considering the revelation principle (Myerson, 1979, 1981), finding a direct incentive-compatible (IC) mechanism $\mathcal{M} : \{\alpha, \mathcal{T}\}$ is without loss of generality.[6] α is the probability to audit an agent and $\mathcal{T}$ the effective tax paid, namely $\mathcal{T}(w^s) = \tau x(w^s)$. The direct IC mechanism is as follows

$$\mathcal{M} = \begin{cases} \mathcal{T} : & [\underline{w}^s, \overline{w}^s] \to [0, \overline{w}^s] \\ \alpha : & [\underline{w}^s, \overline{w}^s] \to [0, 1] \end{cases}$$

A direct IC mechanism in this setting results in the optimal income declaration of every self-employed agent being equal to their real income, $x(w^s) = w^s$. Thus, in equilibrium, evasion does not exist.

Unless the mechanism follows the implementability requirements, some self-employed hide part of their income from the IRS. Let us define $V^s(w^s, \mathcal{M})$ as the indirect utility in sector s given a mechanism $\mathcal{M}$. Any implementable direct IC mechanism has the following characteristics

Lemma 1 *The IC mechanism $\mathcal{M}$ is implementable if and only if*

1. $\alpha(w^s)$ *is non-increasing in* w^s

2. $$\mathcal{T}(w^s) = \frac{w^s(1 - \tau\alpha(w^s)\pi)}{1 - \pi\alpha(w^s)} - \frac{\int_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau)\, dj}{1 - \pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1 - \pi\alpha(w^s)}$$

[6] In the wage-earner sector, taxpayers cannot misreport income; implying that it is not necessary to define a direct IC mechanism there.

Proof.

See Appendix B ■

The above Lemma describes the mechanism's characteristics to incentivize agents to reveal their true income. The first requirement means that, given the incentive to underreport income, audits cannot be increasing in self-employment wages. In the opposite case, the IRS will audit high-income reports more intensively, increasing the probability of non-auditing at low income levels and promoting evasion there. The second requirement establishes the tax amount that incentivizes agents to reveal their true income. This requirement shows the effect of evasion on the tax system and that the IRS affects taxpayers' taxes through the audit level. To promote truth-telling, the IRS should reward some declarations, depending on the audit level. In other words, the IRS can allow smaller income declarations and taxes to incentivize truth-telling in some agents.

If the audit function takes the value of $1/\pi$, self-employed always reveal their real income (Scotchmer, 1987).[7] This result allows redefining the support for audit function as $\alpha(w^s) \in [0, 1/\pi]$.[8]

By using requirements in Lemma 1, it is possible to define the occupational choice rule and the threshold function in the agents' occupational decisions. These elements are helpful in understanding distortions by IRS policy, as well as to simplify the IRS problem.

Definition 1 (Occupational Choice Rule in an IC Mechanism) *For any direct IC mechanism $\mathcal{M}$, $\mathcal{W}^i(\mathcal{M})$ is the set which results from agents' occupational decisions and is defined by*

$$\mathcal{W}^s(\mathcal{M}) = \{(w^s, w^w) \in [\underline{w}^s, \overline{w}^s] \times [\underline{w}^w, \overline{w}^w] \mid V^s(w^s, \mathcal{M}) \geq U^w(w^w, R)\}$$
$$\mathcal{W}^w(\mathcal{M}) = \{(w^s, w^w) \in [\underline{w}^s, \overline{w}^s] \times [\underline{w}^w, \overline{w}^w] \mid V^s(w^s, \mathcal{M}) < U^w(w^w, R)\}$$

Lemma 2 (IC Threshold Function) *For any direct IC mechanism $\mathcal{M}$, there exists a threshold function $\kappa : [\underline{w}^s, \overline{w}^s] \to \mathbb{R}_+$ that establishes the minimum wage to work as a wage-earner. Moreover, in equilibrium the threshold function is characterized by*

$$\kappa'(w^s) = \frac{V^s_{w^s}(w^s, \mathcal{M})}{U^w_{\kappa(w^s)}(\kappa(w^s), R)}$$

where X_i denotes the derivative of variable X with respect to i.

Proof.

See Appendix C ■

Consequently, the IRS incentivizes self-employed agents to reveal their real wages by choosing a direct audit function that maximizes the expected collection and holds conditions in Lemma 1.

[7] To see this, note that if $\alpha(w^s) = 1/\pi$ the utility in the self-employment sector is $U^s(w^s, R) = w^s(1-\tau)$, implying that $x(w^s) = w^s$.

[8] Condition 1 in Lemma 1 gives a restriction on marginal taxes, which is the same as the assumption made earlier:

$$\begin{aligned} 1 - \alpha(w^s)\pi\tau &\geq 0 \\ \frac{1}{\alpha(w^s)\pi} &\geq \tau \end{aligned}$$

The upper limit of the audit function is $1/\pi$, and the marginal tax is limited from above with a decreasing function on audits. This implies that the most restrictive limit is when $\alpha = \frac{1}{\pi}$, which in this case produces $1 \geq \tau$.

Formally, the problem of the IRS is as follows

$$\max_{\alpha(w^s)} \int\limits_{\mathcal{W}^w(\mathcal{M})} \tau w^w dF^w(w^w)dF^s(w^s) \; + \int\limits_{\mathcal{W}^s(\mathcal{M})} \mathcal{T}(w^s)dF^w(w^w)dF^s(w^s)$$

$s.t.$

$$B \geq \int\limits_{\mathcal{W}^s(\mathcal{M})} c\cdot\alpha(w^s)dF^w(w^w)dF^s(w^s)$$

$\alpha(w^s)$ non-increasing in w^s

$$\mathcal{T}(w^s) = \tau x(w^s) = \frac{w^s(1-\tau\alpha(w^s)\pi)}{1-\pi\alpha(w^s)} - \frac{\int\limits_{j=\underline{w}^s}^{w^s}(1-\alpha(j)\pi\tau)\,dj}{1-\pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1-\pi\alpha(w^s)}$$

$$U^w(\kappa(w^s),R) = V^s(w^s,\mathcal{M})$$

The first restriction is the budget constraint, which asserts that the IRS audits until it equalizes the expected audit cost with its budget. The second and third conditions are the implementability requirements for the mechanism $\mathcal{M}$. Finally, the fourth condition is the threshold function's definition, which defines the occupational choice rule. Also, since the IC direct mechanism is used, the IRS function does not have penalties and uses the effective tax instead.

2.3 Government

The government chooses the marginal tax rate $\tau \in [0,1]$, a public goods provision $R \in \mathbb{R}_+$, and the budget for the IRS $B \in \mathbb{R}_+$, to maximize a social welfare function (SWF). Let us define $G(U)$ as the SWF, and assume that $G' > 0$ and $G'' \leq 0$. In this case, G depends on the agent's utility instead of their sum. This assumption captures the government's concern for each possible income realization, obtaining a generalized social weight (Saez and Stantcheva, 2016).

The government has a budget constraint where the cost of providing one unit of a public good is one. With this fact, the problem of the government is as follows

$$\max_{\tau,R,B} \int\limits_{\mathcal{W}^w(\mathcal{M})} G\left(U^w(w^w,R)\right) dF^w(w^w)dF^s(w^s) + \int\limits_{\mathcal{W}^s(\mathcal{M})} G\left(V^s(w^s,\mathcal{M})\right) dF^w(w^w)dF^s(w^s)$$

$s.t.$

$$\int\limits_{\mathcal{W}^w(\mathcal{M})} \tau w^w dF^w(w^w)dF^s(w^s) \; + \int\limits_{\mathcal{W}^s(\mathcal{M})} \mathcal{T}(w^s)dF^w(w^w)dF^s(w^s)$$

$$+\left(B - \int\limits_{\mathcal{W}^s(\mathcal{M})} c\cdot\alpha(x(w^s))dF^w(w^w)dF^s(w^s)\right) \geq B + R$$

$$x(w^s) \in arg\max_{\tilde{x}(w^s)}\{U^s(w^s,R)\}$$

$\{\alpha, \mathcal{T}\} \in \mathcal{M}$ that solve the problem of the IRS

$$U^w(w^w,R) \geq 0,\; V^s(w^s,\mathcal{M}) \geq 0$$

The first government restriction is the budget constraint, which says that the expected tax collection must be at least equal to the expenses in the public goods provision and the budget for the IRS. This means that the government decides the IRS's resources or its size. Later restrictions are associated with the optimal direct IC mechanism that solves the problem of the IRS. Finally, the restriction is limited liability, which states that any agent has a non-negative utility.

2.4 Full-Information Solution

The government decides the sector in which each agent works and its consumption level. In this setting, each agent's productivity is used instead of their wages. Therefore, the government chooses a consumption function, $C : [\underline{n}^s, \overline{n}^s] \times [\underline{n}^w, \overline{n}^w] \to \mathbb{R}_+$, a public goods provision $R \in \mathbb{R}_+$, and an occupational choice function, $Z : [\underline{n}^s, \overline{n}^s] \times [\underline{n}^w, \overline{n}^w] \to \{s, w\}$. The occupational choice function determines the sector where each agent will optimally work.

Proposition 1 *The first-best solution in this setting is given by* $\{\mathcal{Z}_s^*, \mathcal{Z}_w^*,\ R^*, C_i^*(n^i, n^j)\}$*, with* $i \neq j$ *and* $i, j \in \{s, w\}$*, and takes the following form*

$$
\begin{aligned}
\mathcal{Z}_w^* &= \{w \mid \forall (n^s, n^w) \in \mathcal{Z}_w^* \ \Rightarrow\ n^s < n^w\} \\
\mathcal{Z}_s^* &= \{s \mid \forall (n^s, n^w) \in \mathcal{Z}_s^* \ \Rightarrow\ n^s \geq n^w\} \\
1 &= \int\limits_{\mathcal{Z}_w^*} MRS(R^*, C_w^*) dF^w(n^w) dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} MRS(R^*, C_s^*) dF^w(n^w) dF^s(n^s) \\
R^* &= \int\limits_{\mathcal{Z}_w^*} \left(n^w - C_w^*(n^s, n^w)\right) dF^w(n^w) dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} \left(n^s - C_s^*(n^s, n^w)\right) dF^w(n^w) dF^s(n^s) \\
&\frac{dG\left(u(C_w(n^s, n^w), R)\right)}{du(C_w(n^s, n^w), R)} = \frac{dG\left(u(C_s(n^s, n^w), R)\right)}{du(C_s(n^s, n^w), R)}
\end{aligned}
$$

where MRS *is the marginal rate of substitution and*

$$
MRS(R^* C_i^*) = \left. \frac{du/dR}{du/dC_i} \right|_{R^*, C_i^*}
$$

Proof.
See Appendix D ■

Proposition 1 shows the first-best solution in this setting. The first and second equations show that each agent works in its most productive occupation. The next equation reflects the optimal provision of public goods, showing that the marginal cost of providing the public goods equals the sum among agents of the marginal rate of substitution between the public and consumption goods. This result is known as the Bowen-Lindahl-Samuelson (henceforth BLS) rule (Samuelson, 1954, 1955). The last two equations relate to the optimal consumption choice, meaning that two agents in different occupations with the same income should have the same consumption. This establishes a *equality* condition in this setting related to horizontal equality between occupations.

3 IRS Problem

Some procedures are performed to solve the IRS problem. Firstly the audit function is replaced using the IC condition, ${dV^s(w^s,\mathcal{M})}/{dw^s} = V^s_{w^s}(w^s,\mathcal{M}) = 1-\pi\alpha(w^s)\tau$. Secondly, replace the effective tax using the equality $\mathcal{T}(w^s) = w^s - V^s(w^s,\mathcal{M})$. With these changes, it is possible to formulate the problem of the IRS as an optimal control problem, using $V^s(w^s,\mathcal{M})$ as the state variable, and $V^s_{w^s}(w^s,\mathcal{M})$ as the control variable.[9] This specification requires replacing the condition over audits for a condition over the marginal indirect utility, whose must be non-decreasing and its support is $[1-\tau,1]$. By using those changes, the Lagrangian of this problem is

$$\begin{aligned}\mathcal{L}(V^s(w^s,\mathcal{M});V^s_{w^s}(w^s,\mathcal{M})) = &\int\limits_{\underline{w}^s}^{\overline{w}^s}\int\limits_{\kappa(w^s)}^{\overline{w}^w} \tau w^w dF^w(w^w)dF^s(w^s)\\ &+\int\limits_{\underline{w}^s}^{\overline{w}^s}\left[w^s - V^s(w^s,\mathcal{M})\right]F^w(\kappa(w^s))f^s(w^s)dw^s + p(w^s)V^s_{w^s}(w^s,\mathcal{M})\\ &-\mu\left(\int\limits_{\underline{w}^s}^{\overline{w}^s} c\cdot\left[\frac{1-V^s_{w^s}(w^s,\mathcal{M})}{\pi\tau}\right]F^w(\kappa(w^s))f^s(w^s)dw^s - B\right)\end{aligned}$$

where $p(w^s)$ is the adjoint function associated with the state variable, and μ the Lagrange multiplier associated with the budget constraint. The necessary conditions to solve this problem are the following

1. $\dfrac{d\mathcal{L}^*(V^s(w^s,\mathcal{M});V^s_{w^s}(w^s,\mathcal{M}))}{dV^s_{w^s}(w^s,\mathcal{M})} = p(w^s) + \mu\left(\int\limits_{\underline{w}^s}^{\overline{w}^s}\left[\dfrac{c}{\pi\tau}\right]F^w(\kappa(w^s))f^s(w^s)dw^s\right)$
2. $\dfrac{dp(w^s)}{dw^s} = -\dfrac{d\mathcal{L}^*(V^s(w^s,\mathcal{M});V^s_{w^s}(w^s,\mathcal{M}))}{dV^s(w^s,\mathcal{M})} = \int\limits_{\underline{w}^s}^{\overline{w}^s} F^w(\kappa(w^s))f^s(w^s)dw^s$
3. $V^s(\overline{w}^s)$ is free, implying that $p(\overline{w}^s) = 0$.
4. $\mu \geq 0$, $\mu\left(\int\limits_{\underline{w}^s}^{\overline{w}^s} c\cdot\left[\dfrac{1-V^s_{w^s}(w^s,\mathcal{M})}{\pi\tau}\right]F^w(\kappa(w^s))f^s(w^s)dw^s - B\right) = 0$

The first necessary condition does not depend on the control variable, showing that the solution is linear on it, and the control variable takes only extreme values at the optimum. The second condition is positive, which indicates that the adjoint function is increasing, and the third condition suggests that it comes from the negatives to zero. Those three conditions reflect that the budget constraint (condition four) is crucial for identifying the solution. If the budget is enough to audit the entire self-employment sector, or $\mu = 0$, the solution is to audit all self-employed; otherwise, only part of them will be audited. Finally, the solution shows that the IRS always uses its entire budget.

When the IRS is underbudgeted ($\mu > 0$), condition four says that the candidate for a solution is the wage that equalizes the budget constraint. Let us define the wage that produces this equality

[9] Since $\kappa'(w^s) > 0$, it is possible to work with double integrals and simplify some expressions that do not rely on productivity in the wage-earner sector.

as w^*. This level separates the audit function between its extreme values. Below w^*, the IRS audits with maximum intensity, or $\alpha(w^s) = {}^1\!/\!_\pi$. Above w^*, the IRS does not audit or $\alpha(w^s) = 0$. The following equation characterizes w^*.

$$\int_{\underline{w}^s}^{w^*} F^w(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c} \tag{1}$$

Therefore, the IRS spends all its budget auditing with maximum intensity until w^*. The first necessary condition guarantees the existence of this solution since the adjoint function exists, and the second term is always non-negative. Also, since the budget constraint is strictly increasing in auditing, the solution must be unique.

The audit function takes the same form as in Sanchez and Sobel (1993) and fulfills the conditions in Lemma 1. Furthermore, for agents with wages below the threshold, the effective tax is equal to the tax liability designed by the government. On the contrary, agents with wages higher than w^* pay the same tax, which is τw^*.[10] This is because of zero-audit in this zone. In this sense, for those agents, the effective tax takes the form of a poll tax because the marginal effective tax rate equals zero. The following proposition formalizes these results.

Proposition 2 *The direct IC mechanism $\mathcal{M}$ which solves this problem is*

$$\alpha(w^s) = \begin{cases} {}^1\!/\!_\pi & \text{if} \quad \underline{w}^s \leq w^s \leq w^* \\ 0 & \text{if} \quad w^* < w^s \leq \overline{w}^s \end{cases} \tag{2}$$

$$\mathcal{T}(w^s) = \begin{cases} \tau w^s & \text{if} \quad \underline{w}^s \leq w^s \leq w^* \\ \tau w^* & \text{if} \quad w^* < w^s \leq \overline{w}^s \end{cases} \tag{3}$$

where the threshold w^* is defined by

$$\int_{\underline{w}^s}^{w^*} F^w(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c}$$

Corollary 1 *If the IRS has no audit cost (i.e., $c = 0$) the direct IC mechanism $\mathcal{M}$ will be*

$$\alpha(w^s) = {}^1\!/\!_\pi$$
$$\mathcal{T}(w^s) = \tau w^s$$

Proof.

See Appendix E ∎

[10] Note that paying this level of effective taxes is equal to declaring an income equal to w^*.

Figure 2: The IRS Solution

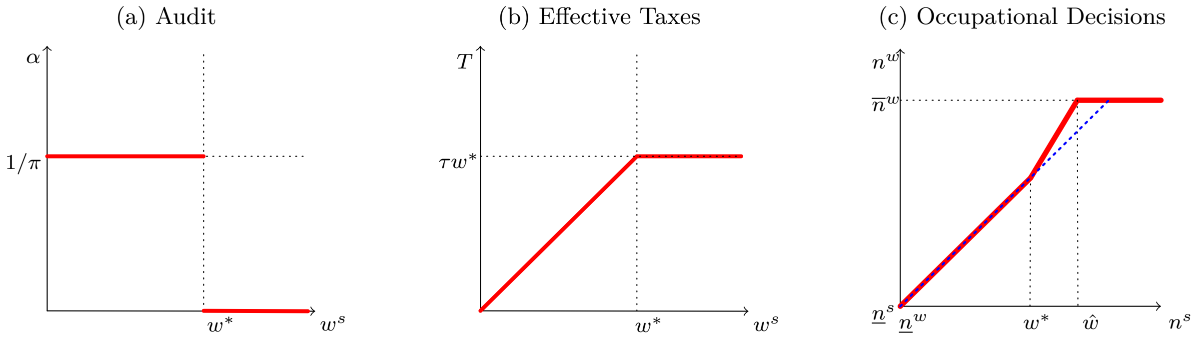


Figure 2 shows the results from Proposition 2. Panel (a) shows that all agents with an income higher than w^* do not face audits. The result of this fact is that, to encourage truth-telling, those agents pay the same amount of taxes, as seen in Panel (b). Finally, the optimal audit and effective taxes result in a distortion in occupational choices for workers with a (potential) self-employment wage higher than the threshold (Panel (c)). Distortions increase the incentive to be self-employed because their after-tax income increases, mainly due to the audit schedule. Therefore, the distortion will vanish if the solution implies auditing the entire self-employment sector.

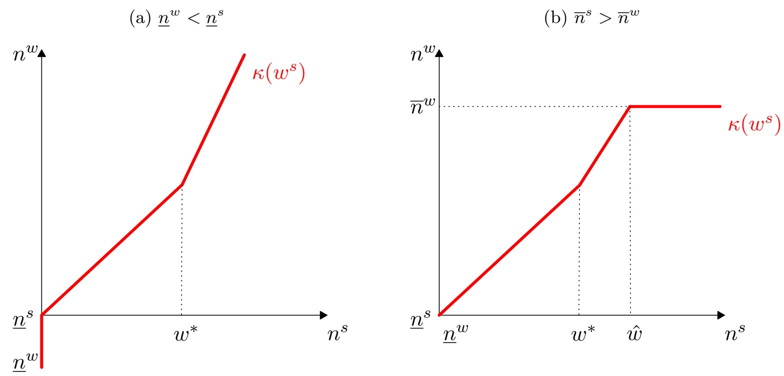
The best action for the IRS is to audit efficiently until w^*. This level depends on three exogenous variables for the IRS: the audit cost, the fine rate, and its budget. If those variables' conjugation implies that it is not optimal to audit the entire self-employment sector, distortions in occupational decisions will appear and come into play only for agents with wages higher than w^*. The result of these distortions is that the higher the marginal tax rate, the more agents decide to become self-employed (see equation (4)). This fact is simple: since the IRS needs to incentivize taxpayers to declare their real wages, the effective tax for non-audited agents is constant. This increases the utility of working as self-employed, provoking distortions in occupational decisions.

4 Government Problem

Before dealing with the government's problem, some relevant aspects of the productivities distribution are clarified. This study focuses on the $\overline{n}^s > \overline{n}^w$ case, as shown in Figure 3 Panel (b).[11] In this case, the existence of a limit in the threshold function, from where its derivative is zero, potentially impacts the optimal results. Another case, in Panel (a) Figure 3, only produces a bigger mass of agents in the full-audit zone. For this reason, the former case is preferred.

[11]This assumption is without loss of generality because Appendix H shows that the solution holds for $\overline{n}^w > \overline{n}^s$. Another paper that follows a similar procedure is Gomes et al. (2017). Finally, this assumption appears natural by including firms or capital owners as self-employed because they self-report their income.

Figure 3: Productivity Distribution Cases



Let us define the point where the threshold function takes the upper-bound of the wage-earner distribution as $\hat{w}$. Given this, the threshold function takes the following form

$$\kappa(w^s) = \begin{cases} w^s & \text{if } w^s < w^* \\ \dfrac{w^s - \tau w^*}{1-\tau} & \text{if } w^* \leq w^s \leq \hat{w} \\ \overline{w}^w & \text{if } \hat{w} < w^s \end{cases} \tag{4}$$

Both instruments, taxes and the IRS budget, modify the shape of the threshold function and the level where it takes the maximum, $\hat{w}$. Any change in these instruments produces two effects: distortions in the incentives to be in some occupation, and changes in the mass of agents that always prefer being self-employed. More specifically, Figure 3 Panel (b) shows that all agents with productivity higher than $\hat{w}$ always prefer being self-employed because their utility in sector s is always greater than in sector w. Finally, w^* can be equal to $\hat{w}$, but this case depends on the budget for the IRS.

The procedure to solve the government problem uses the budget constraint and incorporates the rest of the restrictions into the solutions. Let δ be the Lagrange multiplier for the government budget constraint. The Lagrangian for this problem is

$$\begin{aligned} \mathcal{L} = & \int\limits_{\mathcal{W}^w(\mathcal{M})} G\left(U^w(w^w, R)\right) dF^w(w^w) dF^s(w^s) + \int\limits_{\mathcal{W}^s(\mathcal{M})} G\left(V^s(w^s, \mathcal{M})\right) dF^w(w^w) dF^s(w^s) \\ & - \delta \Bigg\{ B + R - \tau \int\limits_{\mathcal{W}^w(\mathcal{M})} w^w dF^w(w^w) dF^s(w^s) - \tau \int\limits_{\underline{w}^s}^{w^*} w^s F^w\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^* \int\limits_{w^*}^{\hat{w}} F^w\left(\kappa(w^s)\right) dF^s(w^s) \\ & - \tau w^* \int\limits_{\hat{w}}^{\overline{w}^s} F^w\left(\kappa(w^s)\right) dF^s(w^s) \Bigg\} \end{aligned}$$

4.1 Optimal Public Goods Provision

To describe the optimal Bowen-Lindhal-Samuelson (BLS) rule, the notation MRS is used for the marginal rate of substitution between the public and consumption goods. The following proposition formalizes the optimal BLS rule in this model.

Proposition 3 *The Bowen-Lindahl-Samuelson rule is*

$$\phi'(R) = \int\limits_{\mathcal{W}^w(\mathcal{M})} MRS(RC)dF^w(w^w)dF^s(w^s) + \int\limits_{\mathcal{W}^s(\mathcal{M})} MRS(RC)dF^w(w^w)dF^s(w^s) = 1 \quad (5)$$

Proof.
See Appendix F.1 ■

The BLS rule is the same as the first-best. Since agents' utility is separable in consumption and public goods, benefits from consumption are linear, and the labor supply is fixed, the MRS does not change (Boadway and Keen, 1993). Hence, even though evasion alters agents' consumption, the quasi-linear form of the utility and the labor supply assumption result in variations in consumption due to evasion not generating distortions in the MRS.[12]

This result implies two relevant conclusions for this study. Firstly, the government has only two instruments to improve social welfare: the marginal tax rate and the IRS budget. Secondly, given the audit solution, some agents pay fewer taxes than without evasion but enjoy the same public goods provision. This difference distorts the distribution of the burden of financing the provision of public goods. Wage-earners and audited self-employed increase their burden while non-audited agents decrease it.

4.2 Optimal Marginal Tax Rate

In order to give a better interpretation, let us define $g(w)$ as the social value of consumption for an agent with income w expressed in terms of the public funds (Saez, 2001; Saez and Stantcheva, 2016). Formally, $g(w) = {}^{(G' \times U'_C)}/_{\delta}$, and, by definition, the average of g is equal to one. This expression indicates the importance of an increase in the agent's consumption concerning public funds.

Proposition 4 *The optimal marginal tax rate is characterized by*

$$\begin{aligned}
&\frac{\tau}{1-\tau}\int\limits_{w^*}^{\hat{w}}(w^s - w^*)\kappa_\tau(w^s)f^w(\kappa(w^s))dF^s(w^s) = \int\limits_{\mathcal{W}^w(\mathcal{M})}(1-g(w^w))w^w dF^w(w^w)dF^s(w^s) \\
&+\int\limits_{\underline{w}^s}^{w^*}\int\limits_{\underline{w}^w}^{\kappa(w^s)}(1-g(w^s))w^s dF^w(w^w)dF^s(w^s) + \int\limits_{w^*}^{\hat{w}}\int\limits_{\underline{w}^w}^{\kappa(w^s)}(1-g(w^s))w^* dF^w(w^w)dF^s(w^s) \\
&+\int\limits_{\hat{w}}^{\overline{w}^s}\int\limits_{\underline{w}^w}^{\kappa(w^s)}(1-g(w^s))w^* dF^w(w^w)dF^s(w^s) \quad (6)
\end{aligned}$$

[12] For the case that labor supply is not fixed, Blomquist et al. (2016) demonstrate that this result changes. Tax evasion affects the MRS through its effect on labor supply, with the result that the BLS rule diverges from the first-best.

Proof.
See Appendix F.2. ■

The marginal tax rate (MTR) can be decomposed into two effects: welfare and revenue. The welfare effect shows the impact of change in the agent's utility through the effect of taxes on consumption. When taxes rise, all agents face a larger tax liability than before and consume less. However, the reduction in their consumption depends on each consumer's wage. This effect must be measured in public funds, and $g(w)w$ captures it. The revenue effect has two components. First, a rise in taxes increases the marginal tax liability, which is equal to the taxpayer's wage. Hence, $(1-g(w))w$ captures the net monetary effect on agents' welfare. Second, an increase in taxes causes some agents with income higher than w^* to change their occupation and pay a different tax level. This effect is new and comes from incorporating occupational decisions. When taxes rise, a taxpayer with a former wage $\kappa(w^s)$ moves from the wage-earner to the self-employment sector and pays τw^*. The term which captures this effect is $\tau(\kappa(w^s)-w^*)\kappa_\tau(w^s)f^w(\kappa(w^s))$. This term is new and comes from the introduction of occupational decisions. In equilibrium, the sum of those effects must be zero.

Figure 4: Effect of Raising the Marginal Tax Rate

(a) Agents Distribution (b) Taxes Paid

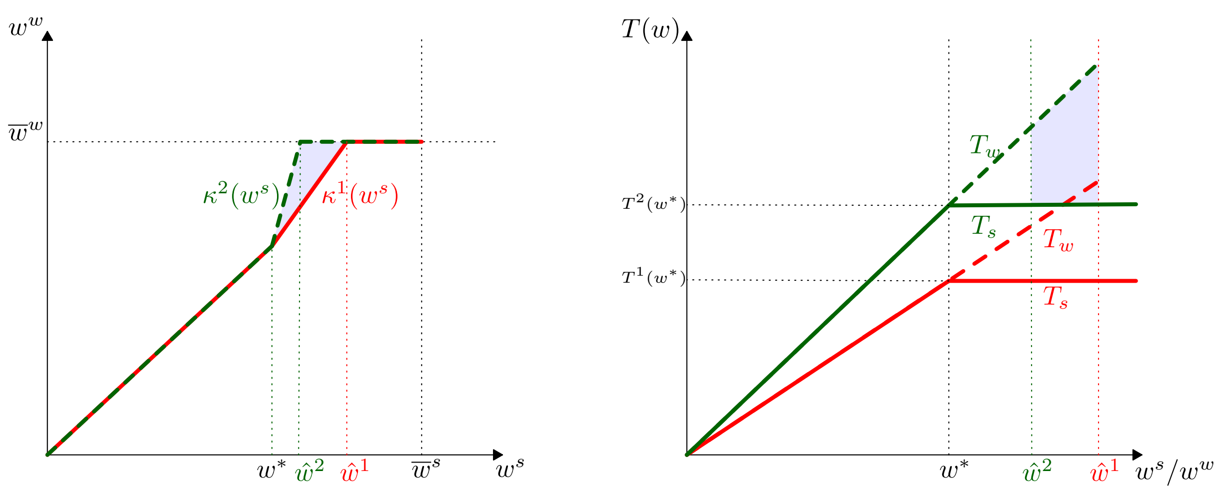


Figure 4 Panel (a) shows the consequences of increasing the MTR on the threshold function and occupational distribution. The red line indicates the initial situation, whereas the dashed green line illustrates a rise in the marginal tax. All agents with a productivity tuple below the red (green) line (dashed line) are self-employed. An increase in taxes results in agents with productivity between w^* and $\hat{w}^1$ changing their occupational choices, which is depicted by the blue area. However, w^* does not change because neither does the IRS budget.[13] These results yield an increase in the mass of self-employed agents. Additionally, an increase in the marginal tax rate causes the productivity

[13]To portray this fact, let us differentiate the threshold function with respect to τ and w^* and equalize it to zero:

$$\int_{\underline{w}^s}^{w^*} \left(\frac{\partial \kappa}{\partial w^*}dw^* + \frac{\partial \kappa}{\partial \tau}d\tau\right) f^w(\kappa(w^s))dF^s(w^s) + F^w(w^*)f^s(w^*)dw^*dw^s = 0$$

level $\hat{w}$ to fall, from $\hat{w}^1$ to $\hat{w}^2$. This change increases the mass of agents that always prefer to be self-employed. The effects are twofold. Firstly, a collective of agents changes their occupation, modifying the proportion of agents in each occupation. Secondly, agents who change their occupation face new taxes, impacting the tax revenue and their utility.

Figure 4 Panel (b) shows the consequences of an increase in marginal tax on the taxes paid by agents and tax revenue. The red line represents the initial situation, and the green line refers to a rise in taxes. The solid line is for taxes paid in the self-employment sector, and the dashed line indicates wage-earners' taxes. When taxes rise, all agents pay higher taxes than before, represented by the difference between the red and green lines, increasing tax revenue. However, some agents change their occupations and face fewer taxes. The agents who change their occupation are those who do not face audits, hence, they bear taxes equal to τw^* instead of τw^w. This change produces a revenue loss for the government, depicted by the blue zone. Moreover, the more taxes increase, the greater the revenue loss. Therefore, tax revenue rises and welfare loss, adding to the occupational effect on revenue (decrease) and welfare (increase), are the forces behind the optimal tax rate.

The inclusion of occupational choices results in the optimal MTR being smaller than one. This result is opposed to Sanchez and Sobel (1993), who obtain a tax rate equal to one, showing that not considering occupational decisions induces an upward bias in the tax rate. This is due to the forces behind the welfare and revenue effects, which go in opposite directions. Raising taxes mechanically increases tax collection but also reduces agents' welfare. This yields a movement of agents to self-employment to pay fewer taxes and increase their utility, eroding tax revenue. This finding is evidence for the forces behind the Laffer curve. The marginal tax will increase if increases in total revenue and welfare by occupational movement are more significant than welfare and revenue losses due to occupational changes. Otherwise, the marginal tax will decrease.

4.3 Optimal IRS Budget

The budget for the IRS delimits the size of the tax authority. In this sense, the choice of this amount is not only a cost but also has repercussions for compliance policy and agents' decisions. Mainly, changes in the IRS budget induce changes in occupational choices and tax revenue. These are the forces behind the optimal budget for the IRS.

Proposition 5 *The optimal budget for the IRS is characterized by*

Dividing both sides by $d\tau$ yields:

$$\int_{\underline{w}^s}^{w^*} \left(\frac{\partial \kappa}{\partial w^*} \frac{dw^*}{d\tau} + \frac{\partial \kappa}{\partial \tau} \right) f^w(\kappa(w^s)) dF^s(w^s) + F^w(w^*) f^s(w^*) \frac{dw^*}{d\tau} dw^s = 0$$

The only way to obtain $\frac{dw^*}{d\tau} = 0$ is if $\frac{d\kappa}{d\tau} = 0$. This comes from the IRS's solution, which says that $\kappa(w^s) = w^s$ for $w^s < w^*$ and $\kappa(w^*) = w^*$ for $w^s = w^*$.

$$\tau \int_{w^*}^{\hat{w}} (w^* - \kappa(w^s)) \frac{dw^*}{dB} \kappa'_B f^w(\kappa(w^s)) dF^s(w^s) = 1 + \int_{w^*}^{\hat{w}} \int_{\underline{w}^w}^{\kappa(w^s)} (g(w^s) - 1) \tau \frac{dw^*}{dB} dF^w(w^w) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^w}^{\kappa(w^s)} (g(w^s) - 1) \tau \frac{dw^*}{dB} dF^w(w^w) dF^s(w^s) \quad (7)$$

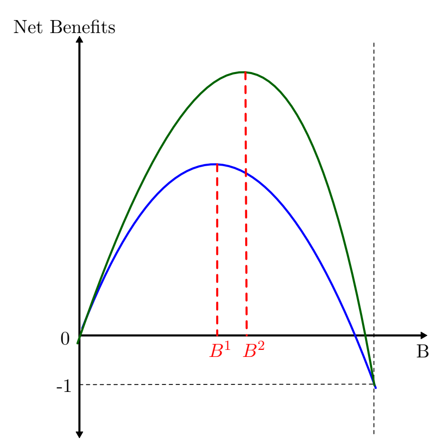
Proof.

See Appendix F.3. ■

It is possible to decompose the budget into three effects: behavioral, mechanical, and welfare. The behavioral effect relates to the capacity to audit more (fewer) taxpayers, producing changes in agents' occupational decisions. This effect is new and comes from the inclusion of occupational decisions. The term that captures the behavioral effect in tax revenue is $\tau(w^* - \kappa(w^s))$. As for the mechanical effect, this reflects the impact on non-audited agents' taxes ($\tau \frac{dw^*}{dB}$) and the cost of changing the budget, which is equal to one. Since the threshold level has risen, taxpayers who do not face an audit pay more taxes than before, which is purely mechanical. This produces a fall in their consumption, captured by the budget effect on taxes, and also produces a welfare loss, which must be measured in terms of public funds ($\tau \frac{dw^*}{dB} g(w)$). In equilibrium, the sum of the three effects must be zero, which provides the characterization in Proposition 5.

When the government gives a larger budget to the IRS, the audit threshold w^* rises. This means that more agents are audited and increases the zone where agents decide their work sector efficiently. Revenue increases because those agents pay higher taxes as wage-earners than as self-employed, and the tax paid in the zero-audit zone also rises since w^* is larger than before. Nevertheless, this also produces a welfare cost due to those agents consuming less. Hence, behind the optimal budget are revenue raised from the behavioral and mechanical effects and costs from welfare and mechanical effects from the increase in the government budget.

Figure 5: Optimal IRS Budget Comparison



What is novel in this derivation is the behavioral effect on occupational decisions. Since the government has a new channel to impact agents' behavior, it is worth its while to provide more resources to the IRS. Figure 5 shows this, where the blue line is the net benefit curve without occupational decisions and the green line when occupational choices are considered. The optimal budget B^2 is larger than without considering occupational decisions B^1. This is because the behavioral effect on occupational decisions is positive, and the marginal cost of increasing the budget does not change compared to the case without occupational choices. In other words, because of more vigorous audits, agents are incentivized to be wage-earners at the margin, and the government can increase its revenue through this. However, this increase in the budget cannot attain the level of auditing all self-employed since the marginal cost lower-bound is one, and the marginal benefit is zero at this point.

The optimal budget shows the relevant results. First, like in Sanchez and Sobel (1993), the IRS always has a smaller budget than would allow it to audit the entire self-employment sector. This is because the lower bound of the marginal cost for increasing the budget is always one and the marginal gains are zero at that budget level. Second, in determining the optimal budget, it is necessary to consider the effect on welfare. This result produces a distance from the cost-benefit analysis. Third, as established in Slemrod and Yitzhaki (1987), the government must consider the effect on the agents' tax burden when deciding the IRS budget. Finally, the budget is larger than the case without occupational decision because of the marginal gains due to the behavioral effect.

5 Equilibrium

The equilibrium in this model implies that the provision of a public good is equal to the first-best, the marginal tax rate is smaller than one, and the budget for the IRS is insufficient to audit all self-employment sector. Hence, the IRS is underbudgeted, such that high-income self-employed are not audited and pay a flat effective tax, distorting their occupational choice in favor of self-employment. Therefore, the tax system becomes regressive due to the incentives for truth-telling in the self-employment sector.[14]

At the optimum, relevant distortions exist that can explain some empirical facts. Firstly, although the tax administration policy is optimal and impedes evasion, agents distort their occupational decisions in favor of self-employment. The connection between taxes and self-employment decisions has been studied extensively, showing a positive relationship.[15] However, the effect of evasion is always behind this fact. Parker (2003) tries to capture the evasion effect, finding no effect, but Castillo and Safojan (2022) find a significant effect exploiting a particular setting that isolates the evasion channel.

The forces behind the model's equilibrium distortions help explain this evidence. The further the tax administration policy is from the optimum, the more extensive the distortions in occupational decisions, and this comes from the effect of taxes on evasion. Higher taxes and weak tax compliance capacity promote self-employment by increasing evasion's marginal gains and facilitating it. The lack of resources to audit all self-employed induces the appearance of this mechanism. If the IRS

[14] Cremer et al. (1990); Erard and Feinstein (1994) and Reinganum and Wilde (1986) obtain similar theoretical results about the regressivity of the system because of the existence of tax evasion.

[15] See, for instance, Bárány (2019); Bosch and de Boer (2019); Bruce (2000); Cullen and Gordon (2007); Fossen and Steiner (2009); Gentry and Hubbard (2000); Schuetze (2000) and Wen and Gordon (2014).

has enough resources, audits generate an efficient occupational choice, but this is not the case at the optimum. Hence, the zero-audit result generates gains through evasion, and this is critical to understanding the relationship between taxes and self-employment decisions.

This work sheds light on how to improve these distortions. First, and in line with Jung et al. (2022), the budget for the IRS should be larger because of the marginal gains from agents' occupational decisions. This policy produces an increase in revenue, reducing distortion in evasion and occupational decisions and improving the regressivity of the tax system. Second, the IRS must use audits in two ways: to enforce tax compliance and as a threat to avoid income misreporting. In a normative sense, and in line with Kuchumova (2017), audits should be stronger at those income levels where it is likely to find evaders. Also, Almunia and Lopez-Rodriguez (2018) show that firms make strategic income declarations to avoid more forceful audits. Hence, audits must be used as a tool for tax compliance and a threat to strategic behavior. In this case, the agent's incentive is to underreport income to pay fewer taxes. For this reason, audits must be more common at low- and middle-wage levels.[16]

Also, the relationship between the marginal tax rate and the threshold function may explain the empirical relation between tax rates and income tax evasion. If the marginal tax falls, the threshold function will take smaller values, reducing the incentive to evade taxes and resulting in less incentive to be self-employed. Empirically, Berger et al. (2016), Castillo and Safojan (2022), and Kleven et al. (2011) show a positive relationship between tax rates and evasion. This relationship indicates that the government must reduce incentives for evasion by cutting taxes or increasing the IRS budget. Along this line, this paper demonstrates that occupational decisions produce lower optimal taxes and a larger budget for the IRS. This mechanism is similar to previous findings in the general equilibrium literature (Kesselman, 1989; Watson, 1985), and reinforces the importance of taxes as a tool in discouraging tax evasion. However, taxes cannot be zero. The critical issue behind this result is audit distortion on occupational decisions from the lack of resources for the IRS. Therefore, it is necessary to align all the tax administration policy to impede and disincentivize evasion and improve the labor market's efficiency.

Distortions of occupational decisions also distort the *equality* condition over consumption in Proposition 1. Behind this are other elements analysed before, particularly the effective tax. However, this sheds light on the evasion effect of inequality captured by consumption. Because of the incentives to declare their true income, some agents are gifted with smaller effective taxes, increasing their consumption compared to similar wage-earners. This affects horizontal equality but also distorts the distribution of appreciated incomes. This is a problem from two perspectives. First, evasion distorts the optimal allocation of consumption and works against horizontal equality. Secondly, through an increase in consumption, some agents increase their welfare. This could imply a Pareto improvement if evasion does not produce an increase in taxes.[17] Therefore, evasion can distort consumption distribution, increasing inequality but improving welfare, even being a Pareto improvement.

[16]If low-productivity agents have the motivation to report greater productivity or income, Bigio and Zilberman (2011) and Zilberman (2016) show that audits increase with the agents' productivity, producing the opposite result to the one shown in the equation 2.

[17]To see a similar conclusion, but in another context, refer to Canta et al. (2020).

Finally, agents have greater incentives to be self-employed at the equilibrium, affecting production efficiency. If the system provides incentives to move resources from one sector to another, there would be a lack of productivity where the resources were first. In other words, firms would not find enough high-skilled workers in the wage-earner sector because of their move to self-employment. This contradicts the Diamond-Mirrlees theorem (Diamond and Mirrlees, 1971a,b) in the sense that efficiency in productivity is not attained in the second-best. Gomes et al. (2017) find a similar conclusion in a model without tax evasion and IRS, and Best et al. (2015) find empirical evidence of the gains from inefficient productivity policies for improvement in tax compliance and tax revenue. In this case, the difference arises because it is not optimal to provide enough resources to the IRS to conduct audits in the self-employment sector.

This discrepancy, as established in Best et al. (2015), is based on the differences in tax capacity. Environments with lower tax capacities could benefit from inefficient productivity policies if those increased tax compliance and tax collection. This paper theoretically demonstrates two results straightforwardly. First, evasion is a critical issue that must be distanced from efficiency in production. Taxpayers choose occupations or sectors seeking to increase their utility, and if some sector presents higher evasion opportunities, distortions appear in the optimal work sector choices. Secondly, a joint strategy between institutions (IRS and Governments) and policies (tax schemes, audits, penalties, among other things) is necessary to recover productivity efficiency. This is because, at the optimum, distortions in occupational decisions exist, and a joint strategy is needed to solve them.

6 Discussion

Although the equilibrium in this model has already been shown, some questions are still open. Firstly, this section looks into imposing two different linear tax rates, one for each occupation. This extension could help to recover efficiency in allocating workers. Lastly, the problem of achieving the same audit result with non-linear audit costs and imposing an increasing fine rate is resolved.

6.1 Differential Taxation

Differential taxation allows the seeking of efficiency in allocating agents, even if the audit schedule produces incentives to be self-employed. For this approach, the assumption is that both marginal tax rates cannot be higher than one. First, let us define the threshold function under this approach.

$$\kappa(w^s) = \begin{cases} \dfrac{w^s(1-t_s)}{1-t_w} & \text{if } w^s < w^* \\ \dfrac{w^s - t_s w^*}{1-t_w} & \text{if } w^* \leq w^s < \hat{w} \\ \overline{w}^w & \text{if } \hat{w} \leq w^s \end{cases}$$

where t_s and t_w are the tax rates in the self-employment and the wage-earner sector respectively. Although this specification changes the threshold function, it does not alter the form of the optimal audit. This result comes from the necessary conditions for solving the IRS problem. Differential taxation only changes the applicable tax rate to determine the threshold. This is to say, audit the entire self-employment sector or until w^*.[18] For this reason, the same optimal audit pattern is valid for this specification.

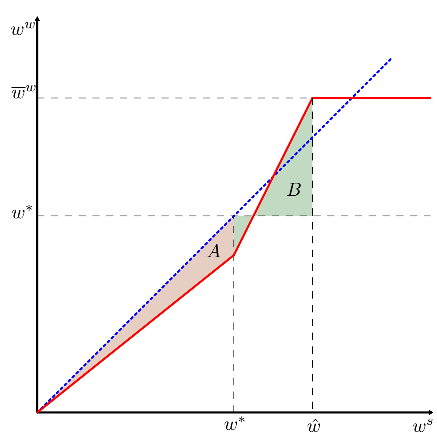
The government solves a Lagrangian similar to that in Section 4, choosing both tax rates. As previously explained, the provision of public goods is the same as in the first-best. This result is maintained in this framework because choosing two different tax rates does not alter the reason that supports it, risk-neutrality. Also, since the focus is only on differential taxation, the optimal budget for the IRS is not obtained.

Proposition 6 *In a hierarchical model with occupational choices where the IRS maximizes expected tax collection and the government maximizes social welfare, differential taxation implies a higher marginal tax rate in the sector where evasion is possible. Moreover, differential taxation maximizes social welfare, being a Pareto improvement from the situation with one equal marginal tax rate for both sectors.*

Proof.
See Appendix G. ■

Figure 6 shows the basic idea behind Proposition 6. The red line represents the threshold function, and the blue dotted line represents the same wage in both sectors. Zone A represents revenue losses from agents who decide to be wage-earners but pay higher taxes as self-employed. In contrast, zone B shows the increase in tax collection from agents who pay higher taxes in the wage-earner sector than in the self-employment sector. Differential taxation is possible only if zone A is smaller than zone B. Thus, differential taxation occurs if the increase in tax collection from agents above w^* is greater than the losses from agents below it. When the difference in the marginal taxes rises, the threshold function falls, increasing A and decreasing B. Thus, it is not optimal for the government to separate both marginal taxes greatly. Consequently, differential taxation aims to solve tax collection losses due to non-audits on the top rather than improving the allocation of agents. This ends up distorting the entire worker population's occupational decisions even more.

Figure 6: Differential Taxation



[18]To see this, note that in the Lagrangian from the problem of the IRS, marginal indirect utility is used as a control variable. Thus, necessary conditions 2 and 3 do not change. Only condition 1 changes, using t_s instead of τ, and this does not change the form of the solution.

A higher tax in the self-employment sector goes opposite to the traditional literature in optimal taxation, which recommends lower taxes for the self-employed. However, despite the differences in policy recommendations, the meaning of both arguments is the same: using tax incentives to eliminate the motivation to evade. When occupational decisions are incorporated into the optimal taxation model, a rise in the self-employment tax rate eliminates the incentive to evade, and agents move to the wage-earner sector. For this reason, both arguments go in the same direction but with different recommendations.

The result of two different tax rates shows that dividing the tax system regarding the different occupations is a Pareto improvement. The reason comes from the evasion effect and the use of taxes to discourage it. Since evasion affects the marginal gains from evading, disparities in evasion facilities should be treated one by one depending on the context. This results in tax policy going in line with audits, discouraging evasion and moving resources to occupations where evasion does not exist.

However, differential taxation produces a worse allocation of workers than the former setting, producing a puzzle regarding production efficiency. As was pointed out, distortions in occupational decisions contradict the Diamond-Mirrlees theorem. Hence, this extension does not resolve this contradiction; instead, differential taxation makes the difference deeper. This result is produced by the linear tax system and the usefulness of moving taxpayers to the wage-earner sector. First, the linear tax assumption makes the government distort all agents to resolve problems in the zero-audit zone. Second, incentivizing workers to be wage-earners helps to solve evasion and can increase welfare, which is a good policy for the government.[19] In addition, this result also provides evidence that a limited tax compliance context would benefit from distortionary policies regarding productivity, similar to Best et al. (2015). This arises because separating the tax system produces an increase in revenue and welfare but increases the inefficiency in productivity.

Corollary 2 *In a hierarchical model with occupational choices and linear tax schemes, it is impossible to recover the efficiency of worker allocation.*

Corollary 2 reflects that it is impossible to reconcile incentives for taxpayers to tell the truth with the incentive to choose their occupations efficiently. This result is essential for designing tax and audit policies. The possible solution for the audit scheme's inefficiencies through differential taxation allows the government to improve economic efficiency and raise revenue. Hence, the government needs to find other policies to improve efficiency and increase revenue.

6.2 Audit Results

6.2.1 Audit Costs

This section extends the analysis over the IRS solution, allowing that the audit cost depends on the agents' wages, and it is known to the IRS when the agent reveals its type.[20] The audit cost is

[19] Regarding this point, Gomes et al. (2017) show the optimality of moving taxpayers to a specific work sector in a non-linear tax setting without evasion. Therefore, the welfare gains from this kind of policy against evasion could exist even in non-linear tax policy models.

[20] Since we draw a parallel between productivity and wages, this assumption is similar to assuming that the audit cost depends on agents' productivity.

defined as $c(w^s)$. From Section 3, the condition that determines the form of the solution is the first necessary condition from the Lagrangian. In this condition, only the audit cost changes because neither the control variable nor the state variable changes.

A condition similar to a single-crossing is found because the above function must cross the zero line once. To obtain this condition, let us derive the new first necessary condition regarding the self-employment wage and use two facts: the second tax derivative is zero, and the derivative of the adjunct function is positive. The following equation establishes the condition over the threshold function.

$$-\frac{c'(w)}{c(w)} < \frac{\pi\tau}{\mu c(w)} + \frac{f^w(\kappa(w))k'(w)}{F^w(\kappa(w))} + \frac{f^{s\prime}(w)}{f^s(w)}$$

where w is the level that produces $\frac{d\mathcal{L}}{dV^s_{w^s}} = 0$. The condition is fulfilled with a positive marginal cost or a strictly increasing cost because the right side is positive by definition. Hence, if the audit cost is monotonically non-decreasing in the self-employment income, the threshold solution is maintained.

6.2.2 Fine Rate

Another interesting result is obtained when the fine rate is not linear. This issue regards the intention to deter tax evasion through high penalties. Two options are analysed: 1) a fine rate increasing with the amount evaded, and 2) a fine rate rising with the real income. These specifications allow the IRS to audit more taxpayers at lower cost because a higher fine rate replaces the audit's deterrence effect, allowing fewer levels.

Given the direct IC mechanism, evasion does not exist in equilibrium, and any instrument that depends on an evaded level vanishes. Thus, a fine rate that depends on the amount of evaded taxes collapses to a linear fine rate in equilibrium. This situation is the same as the one proposed in the model and results in an equal optimal audit, as presented earlier.

As for a fine rate that depends on the real wage, the result is different. In this case, an analysis similar to that realized with audit cost is necessary. Now, the fine rate depends on the self-employment wage.

Deriving the new first necessary condition with respect to the self-employment wage allows us to obtain the single-crossing condition.

$$\frac{\pi'(w)}{\pi(w)} < \frac{\pi(w^s)\tau}{\mu c} + \frac{f^w(\kappa(w))\kappa'(w)}{F^w(\kappa(w))} + \frac{f^{s\prime}(w)}{f^s(w)}$$

As stated before, w is the wage level that produces $\frac{d\mathcal{L}}{dV^s_{w^s}} = 0$. This expression shows that the marginal fine rate must be bounded from above to obtain the threshold form in the audit function. This result allows the imposition of an increasing fine rate but restricts the form that it raises. Therefore, imposing an increasing fine rate to place higher penalties on higher incomes, audit more agents, and improve the IRS's effectiveness is possible.

7 Conclusion

This paper investigates distortions by introducing occupational choices in a hierarchical model similar to that of Sanchez and Sobel (1993). In this sense, these authors' model is extended, making it possible for agents to choose between self-employment and wage-earning. Income tax evasion is possible only in self-employment because agents self-report their income. The focus is on distortions in the audit function, a marginal tax rate, the budget for the IRS, the provision of public goods, and occupational decisions.

The IRS maximizes expected tax collection subject to a budget constraint, with a constant audit cost, and finds a direct incentive-compatible mechanism. Such a mechanism comprises a direct audit and an effective tax, the tax paid by the self-employed who optimally declare their real income. Audits take the maximum value below a threshold and are zero above it. This pattern implies that the IRS must devote all its efforts to auditing sectors in which it is more probable to find evaders. Also, this results in effective tax being the same for self-employed with higher wages than the threshold level. The same result holds with two extensions. First, the audit cost must be monotonically non-decreasing with self-employment income. Second, the fine rate could increase with self-employment income, but its growth must be bounded from above.

The government maximizes social welfare subject to a resource constraint. Because of risk-neutral agents, the optimal public goods provision is the same as in the first-best. The result is that the burden to finance the provision of public goods increases for wage-earners and audited self-employed. The marginal tax rate is smaller than one and comprises two effects: welfare and revenue. The optimal IRS budget is lower than the level that allows auditing all self-employed but higher than the case without occupational decisions. The optimal IRS budget can be characterized by three effects: behavioral, mechanical, and welfare. In this setting, differential taxation is a Pareto improvement and implies a higher marginal tax in the self-employment sector.

Since the IRS is underbudgeted, audits introduce distortions in occupational decisions. Due to institutional policy, the distortions result in agents not working in their most productive sector. This result contradicts the Diamond-Mirrlees theorem because productivity efficiency is not attained in a second-best tax optimum. Moreover, although differential taxation is optimal, this instrument does not solve the distortion of agents' allocation. Differential taxation increases this problem, distorting the allocation of workers even more. This result arises from the necessity to discourage evasion by motivating agents to be wage-earners. Both results show that efficiency in productivity is not attained in settings with low tax capacity, and production-inefficient policies are desirable in this context.

The comparison with Sanchez and Sobel (1993) shows that occupational decisions produce a lower marginal tax rate. Hence, not considering occupational decisions induces an upward tax bias. The upward bias in taxes may increase the distortion in worker allocation because it increases incentives to work as self-employed in those agents who face smaller audits. Therefore, tax design must incorporate the occupational decision to avoid a larger incentive to evade taxes.

Also, occupational decisions produce a larger marginal benefit from increasing the budget for the IRS. Since the IRS could audit more self-employed, tax revenue rises, and the marginal benefits of an extra dollar/euro for the IRS would be larger than the case without occupational decisions. Not considering both sectors produces a lower budget for the IRS, weakening the tax compliance policy, eroding tax collecting, and facilitating evasion. Therefore, it is relevant to consider different occupations because it increases resources for tax compliance policy at the optimum.

The three previously mentioned results show the relevance of considering occupational decisions in tax administration design. If tax administration policy does not consider these agents' choices, distortion in favor of evasion and self-employment appears. This paper demonstrates how the policy should be at the optimum, providing clear policy recommendations. First, audits should be more vigorous where they are more prone to find evaders. After that, supposing all occupations face the same tax scheme, the marginal tax rate should be lower, and the IRS size should be larger than the initial setting without occupational decisions. However, the tax system must not be equal for all occupations. The tax system should differ by occupation, or group those who depend on the facility to evade taxes to attain higher social welfare.

Some problems are still present at the equilibrium, like regressivity in the tax system and inefficiencies in productivity. Even when the government chooses its tax administration policy optimally, this paper demonstrates that distortions exist, and it is necessary to take other courses of action to solve them. Hence, it is essential to inquire about new instruments, mechanisms, and strategies to increase enforcement policies' effectiveness at a lower economic cost. Examples can be focused on persuasion policies that accompany auditing. Since taxes and audits affect expected consumption and occupational choices, a non-pecuniary policy could only affect the last. Indeed, shaming policies (See Perez-Truglia and Troiano (2018) for instance) can be a good example of persuasion policies in the direction of tax compliance.

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Appendix

A Occupational Choice Model vs. Evasion Model

This section demonstrates that incorporating evasion in a differential taxation model is not trivial. To do this, we compare a differential taxation model similar to Gomes et al. (2017) and this paper. Because of its purpose, this section focuses only on the conditions to obtain similar results, not on deriving each of them.

The economy has two sectors, self-employed and wage-earner, s and w respectively. Each agent has an ability tuple $n = (n_s, n_w)$, which represents its productivity in each sector. It is assumed that each productivity comes from a distribution function, twice continuously differentiable, and with a support $[\underline{n}_i, \overline{n}_i]$. Agents choose the labor supply h_i, implying an effective labor $h_i n_i$ in the sector $i \in \{s, w\}$. Labor Income in sector i is $y_i = w_i h_i n_i$, where w_i is the wage rate. The government imposes two different tax schemes, where T_i is the income tax scheme in sector i. Given this, the agent's utility in sector i is

$$u_i(n_i) = w_i h_i n_i - \psi(h_i) - T_i(w_i h_i n_i)$$

where ψ is the convex cost of labor. The following assumption is made to nest this frame with a hierarchical model with tax evasion. In the wage-earner sector, labor supply is inelastic and equals one, that is $h_w = \overline{h}_w = 1$. By using this, the first-order conditions with respect to the labor supply in the self-employment sector and the derivatives with respect to productivity in each sector are

$$\begin{array}{lrcl} \langle h_s \rangle & w_s n_s \left[1 - T_s'(w_s h_s n_s)\right] & = & \psi'(h_s) \\ \langle n_s \rangle & w_s h_s \left[1 - T_s'(w_s h_s n_s)\right] & = & u_s'(n_s) \\ \langle n_w \rangle & w_w \left[1 - T_w'(w_w n_w)\right] & = & u_w'(n_w) \end{array}$$

The threshold function reflects the minimum ability in sector w to be a wage-earner worker, given the agent's ability in sector s. In that sense, it is a threshold level that separates the extensive margin decision. Let $c : [\underline{n}_s, \overline{n}_s] \to [\underline{n}_w, \overline{n}_w]$ be the threshold function. By definition, this function holds with $u_s(n_s) = u_w(c(n_s))$, implying the following condition

$$c'(n_s) = \frac{u_s'(n_s)}{u_w'(c(n_s))} = \frac{w_s h_s \left[1 - T_s'(w_s h_s n_s)\right]}{w_w \left[1 - T_w'(w_w c(n_s))\right]} \tag{8}$$

This function is equal to that in Gomes et al. (2017). Therefore, this results from a model with differential taxation and no evasion.

For the model with tax evasion, let us assume that in the self-employment and wage-earner sectors, the labor supply is inelastic and equal to one. Let x_s be the income report in sector s. In this case, the government decides the audit function and only one income tax scheme for all agents, T. Let $\rho(x_s)$ be the audit function that depends on the declared income in the sector s. If an evader is discovered, it will pay a linear fine on the evaded amount with a penalty rate $\pi > 1$. With these facts, the agent's utility in each sector is

$$\begin{array}{rcl} u_w(n_w) & = & w_w n_w - T(w_w n_w) - \psi(\overline{h}_w) \\ u_s(n_s) & = & w_s n_s - T(x_s) - \rho(x_s)\pi \left[T(w_s n_s) - T(x_s)\right] - \psi(\overline{h}_s) \end{array}$$

With the above definition, both problems are similar because the government chooses two instruments. In the case of evasion, the government chooses the audit probability and the tax scheme, and in the model without evasion, it decides on two tax schemes, one for each occupation.

In the model with evasion, only the derivatives with respect to each sector's ability are obtained. These derivatives are the following

$$\begin{array}{lrcl} \langle n_s \rangle & w_s \left[1 - \rho(x_s)\pi T'(w_s n_s)\right] & = & u_s'(n_s) \\ \langle n_w \rangle & w_w \left[1 - T'(w_w n_w)\right] & = & u_w'(n_w) \end{array}$$

The threshold function in this case is

$$c'(n_s) = \frac{u_s'(n_s)}{u_w'(c(n_s))} = \frac{w_s \left[1 - \rho(x_s)\pi T'(w_s n_s)\right]}{w_w \left[1 - T'(w_w c(n_s))\right]} \tag{9}$$

For the clearest comparison, assume the same wage rate in each model. The critical move is to compare equation 8 with equation 9. The wage-earner sector faces the same conditions in each model. However, since the self-employed face a fine rate and an audit probability in the case of evasion, the situation in the self-employment sector is different.

Let us find the conditions under which both threshold functions will be identical. Suppose that the tax schedule in the model without evasion is optimal for a particular distribution of ability. Since the wage-earner sector is the same in both models, let us define the same tax schedule for each, implying $T = T_w$, and assume that the threshold function c is equal in both model. Otherwise, both models do not fit well since extensive margin decisions will differ. With those assumptions, the denominator in equation 8 and 9 is the same. Now, to obtain the same solution, the same numerators are required. This happens only if the following hold.

$$\rho(x_s) = \frac{1 - h_s \left[1 - T_s'(w_s h_s n_s)\right]}{\pi T'(w_s n_s)}$$

where h_s is the labor supply in the model without evasion. If the above condition holds, both problems produce the same results and are correctly nested. However, if the audit strategy takes other forms, both solutions are not the same. For instance, suppose the following audit function.

$$\rho(x_s) = \begin{cases} 1/\pi & \text{if } n_s < n^* \\ 0 & \text{if } n^* \leq n_s \end{cases}$$

where n^* is the a threshold. This strategy, optimal under a hierarchical model (Sanchez and Sobel, 1993), shows that it is not always possible to obtain the same solution in both models. In Gomes et al. (2017) the optimal tax scheme is non-linear. To obtain the same solution under this form, the requirement should be that $\frac{1-h_s[1-T_s'(w_s h_s n_s)]}{T'(w_s n_s)} = 1$ for $n^s < n^*$ and 0 otherwise. Hence, it is not trivial to assume that both models can be nested. In addition, both problems are different. First, the government faces auditing costs, which produces more expenditure, and consequently a more significant revenue than in the absence of evasion. Second, the existence of evasion is evidence of how imperative it is for the government to consider more incentive-compatible conditions to incentivize agents to declare their real type.

B Proof of Lemma 1

To obtain a direct incentive-compatible mechanism, it is necessary to analyse two restrictions on self-employed, participation (P) and incentive compatibility (IC). In this case, the participation constraint is the limited liability condition, which refers to the non-negative utility for each taxpayer.

$$(P): U^s(w^s, x) = w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi\left[\tau w^s - \mathcal{T}(w^s)\right] \geq 0$$

$$(IC): U^s(w^s, w^s) = w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi\left[\tau w^s - \mathcal{T}(w^s)\right] \geq U^s(w^s, x) = w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi\left[\tau w^s - \mathcal{T}(w^s)\right] \quad \forall x \leq w^s$$

The incentive-compatible restriction can be replaced by the following

$$V(w^s, \mathcal{M}) = \max_x \left\{w^s - \mathcal{T}(x) - \alpha(x)\pi\left(\tau w^s - \mathcal{T}(x)\right)\right\}$$

where $V(\cdot)$ is the indirect utility. Using the envelope theorem, and the optimal condition on the income declaration, we can derive the above expression with respect to w^s. As we seek agents' truth-telling, let us impose this condition, i.e., $x = w^s$, in the derivative.

$$\frac{\partial V(w^s, \mathcal{M})}{\partial w^s} = 1 - \alpha(w^s)\pi\tau$$

Now, by using the fundamental theorem of calculus, let us rewrite the indirect utility for self-employed agents and use its definition to obtain the following equality.

$$\int_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau) + V(\underline{w}^s) = V^s(w^s) = w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi\left(\tau w^s - \mathcal{T}(w^s)\right)$$

where $V(\underline{w}^s)$ is the indirect utility of an agent with income $\underline{w}^s$. By doing algebra we obtain the effective tax formula.

$$\mathcal{T}(w^s) = \frac{w^s(1 - \tau\alpha(w^s)\pi)}{1 - \pi\alpha(w^s)} - \frac{\int_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau)\, dj}{1 - \pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1 - \pi\alpha(w^s)}$$

Now, to demonstrate the first statement, we follow Guesnerie and Laffont (1984). First, we obtain the first-order condition (FOC) for the optimal income declaration in the self-employment sector.

$$\frac{\partial V(w^s, \mathcal{M})}{\partial x} = -\mathcal{T}'(x) - \alpha'(x)\pi(\tau w^s - \mathcal{T}(x)) + \alpha(x)\pi\mathcal{T}'(x) = 0$$

Second, we obtain the second-order condition (SOC). This must be zero or negative because of the quasi-concavity of the utility.

$$\frac{\partial^s V(w^s, \mathcal{M})}{\partial x^2} = -\mathcal{T}''(x) - \alpha''(x)\pi(\tau w^s - \mathcal{T}(x)) + 2\alpha'(x)\pi\mathcal{T}'(x) + \alpha(x)\pi\mathcal{T}''(x) \leq 0$$

Third, the FOC is derived with respect to w^s and assumed truth-telling, i.e., $x = w^s$.

$$\frac{\partial^s V(w^s, \mathcal{M})}{\partial x \partial w^s} = -\alpha'(w^s)\pi\tau + [-\mathcal{T}''(w^s) - \alpha''(w^s)\pi(\tau w^s - \mathcal{T}(w^s)) + 2\alpha'(w^s)\pi\mathcal{T}'(w^s) + \alpha(w^s)\pi\mathcal{T}''(w^s)] = 0$$

The term in brackets is negative or zero from the SOC. This implies that the first term must be positive or zero. Recall that the marginal tax is positive or zero by assumption. Therefore, the audit must be non-increasing in the self-employed wage.

C Proof of Lemma 2

Suppose two agents exist, A and B. Agent A has higher productivity in the wage-earner sector but the same in self-employment compared with agent B. This is $w^w_A > w^w_B$ and $w^s_A = w^s_B$. Therefore both agents have the same utility in the self-employment sector. However, agent A has higher utility in the wage-earner sector than agent B. This is because the utility in the wage-earner sector increases with wage-earner productivity ${}^{dU^w}/_{dw^w} = 1 - \tau \geq 0$. Let us assume that the difference in agent B's utilities means that B is a wage-earner. Since agent A has greater utility in the wage-earner sector than B, it also works in the wage-earner sector. Therefore, every agent with higher productivity in the wage-earner sector and the same productivity in the self-employed sector as agent B is a wage-earner.

Now, let us assume that there is also an agent C, who is less productive in the wage-earner sector and has the same productivity in the self-employment sector as agent B. This is $w^w_B > w^w_C$ and $w^s_B = w^s_C$. Agent C has productivities such that $U^w(w^w_C, R) = U^s(w^s_C, R)$. By Definition 1, agent C is self-employed. Therefore, every agent with lower productivity in the wage-earner sector and the same in self-employment as agent C makes the same occupational decision. This is because utility in the wage-earner sector increases with the wage-earner wage, and every agent with less productivity in the wage-earner sector than C has strictly higher productivity in the self-employment sector.

Therefore for each productivity in the self-employment sector, there exists a productivity in the wage-earner sector, which equalizes both sectors' utility and determines a switch point in the occupational decision. Formally, define this level as κ, and in the example $\kappa(w^s) = w^w_C$. Hence, every agent with $w^w \leq \kappa(w^s)$, works in the self-employment sector.

The second part comes from deriving $V^s(w^s, \mathcal{M}) = U^w(\kappa(w^s), R)$ with respect to w^s.

$$V^s_{w^s}(w^s, \mathcal{M}) = U^w_{\kappa(w^s)}(\kappa(w^s), R)\kappa'(w^s)$$
$$\kappa'(w^s) = \frac{V^s_{w^s}(w^s, \mathcal{M})}{U^w_\kappa(\kappa(w^s), R)}$$

where the expression X_i refers to the derivative of X with respect to i.

D Proof of Proposition 1

The problem that the government solves is as follows

$$\max_{\mathcal{Z}_w,\mathcal{Z}_s,C_w(n^s,n^w),C_s(n^s,n^w),R} \int\limits_{\mathcal{Z}_w} \left[G\left(u(C_w(n^s,n^w),R)\right)\right] dF^w(n^w)dF^s(n^s) + \int\limits_{\mathcal{Z}_s} \left[G\left(u(C_s(n^s,n^w),R)\right)\right] dF^w(n^w)dF^s(n^s)$$

s.t.

$$\int\limits_{\mathcal{Z}_w} n^w dF^w(n^w)dF^s(n^s) + \int\limits_{\mathcal{Z}_s} n^s dF^w(n^w)dF^s(n^s) \geq \int\limits_{\mathcal{Z}_w} C_w(n^s,n^w)dF^w(n^w)dF^s(n^s) + \int\limits_{\mathcal{Z}_s} C_s(n^s,n^w)dF^w(n^w)dF^s(n^s) + R$$

where $\mathcal{Z}_i$ is the set of agents on the work side i and C_i is the consumption function for agents in sector i, with $i \in \{s,w\}$. The government has a resource constraint, which states that production in both sectors must be equal to consumption and spending on public goods.

To prove the first two expressions, suppose that a mass of agents greater than zero is assigned to work in their less productive sector. This mass of agents has $w^s > w^w$ but work in sector w. From the resource constraint, redirecting all these agents to sector s increases economic resources, which can be assigned to increase consumption or the provision of public goods, increasing social welfare. This means that the initial solution cannot be optimal because there exists a reallocation of agents that provides higher social welfare. Therefore, each agent works in the sector with higher productivity.

The fifth expression comes from the maximization of the government problem with respect to the consumption in each sector. The first-order condition (FOC) over consumption is

$$\frac{dG(u(C_i(n^i),R))}{du(C_i(n^i),R)} f^d(n^d)f^s(n^s) - \gamma f^d(n^d)f^s(n^s) = 0, \qquad \text{for } i \in s,w$$

where γ is the Lagrange multiplier for the resource constraint. The fifth expression is obtained by assuming an interior solution and using γ to equalize both conditions.

The third expression comes from the maximization of the social welfare function with respect to public goods. The FOC is

$$\int\limits_{\mathcal{Z}_w^*} \frac{dG}{du} \times \frac{d\phi(R)}{dR} dF^w(n^w)dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} \frac{dG}{du} \times \frac{d\phi(R)}{dR} dF^w(n^w)dF^s(n^s) - \gamma$$

Both sides can be divided by γ and replaced by the FOC from consumption in each sector.

$$\int\limits_{\mathcal{Z}_w^*} \frac{\frac{dG}{du} \times \frac{d\phi(R)}{dR}}{\frac{dG}{du} \times \frac{du}{dC_w}} dF^w(n^w)dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} \frac{\frac{dG}{du} \times \frac{d\phi(R)}{dR}}{\frac{dG}{du} \times \frac{du}{dC_s}} dF^w(n^w)dF^s(n^s) = 1$$

Define $MRS(R^*C_i^*)$ with $i \in \{s,w\}$, as the marginal rate of substitution between public goods and private consumption, evaluated at the optimum. It is critical to note that the MRS is equal for all agents, because of ${}^{du}/_{dC_i} = 1$ and ${}^{du}/_{dR} = \phi'(R)$. This condition gives us the following, which is the third expression.

$$\phi'(R) = \int\limits_{\mathcal{Z}_w^*} MRS(R^* C_w^*) dF^w(n^w) dF^s(n^s) + \int\limits_{\mathcal{Z}_s^*} MRS(R^* C_s^*) dF^w(n^w) dF^s(n^s) = 1$$

Finally, the fourth expression shows the resources needed to finance the optimal provision of public goods R^* and comes from the resource constraint.

E Proof of Corollary 1

If the audit cost is zero ($c = 0$), the first necessary condition for an optimum depends only on the adjoint function. The adjoint function is increasing from the second necessary condition, and by the third necessary condition, it goes from the negative numbers to zero. Hence, the first necessary condition is equal to zero only when the productivity in the self-employment sector takes its upper-bound value, i.e., $w^* = \overline{w}^s$. This means that the IRS audits all the taxpayers in the self-employment sector, and the effective taxes equal the taxes designed by the government.

F Derivation of the Optimal Government Policies

F.1 Optimal BLS Rule

The first-order condition (FOC) with respect to R is

$$\int\limits_{\mathcal{W}^w(\mathcal{M})} \frac{G'\left(U^w(w^w, R)\right)}{\delta} \phi'(R) dF^w(w^w) dF^s(w^s) + \int\limits_{\mathcal{W}^s(\mathcal{M})} \frac{G'\left(V^s(w^s, \mathcal{M})\right)}{\delta} \phi'(R) dF^w(w^w) dF^s(w^s) = 1$$

The FOCs with respect to each agent's consumption in each sector are

$$G'\left(U^w(w^w, R)\right) f^w(w^w) f^s(w^s) - \delta f^w(w^w) f^s(w^s) = 0$$
$$G'\left(V^s(w^s, \mathcal{M})\right) f^w(w^w) f^s(w^s) - \delta f^w(w^w) f^s(w^s) = 0$$

where the definition of consumption, $C(w) = w(1 - \tau)$, is used to replace taxes for consumption. Using the previous equations to replace the Lagrange multiplier in the FOC of the public good yields

$$\int\limits_{\mathcal{W}^w(\mathcal{M})} \phi'(R) dF^w(w^w) dF^s(w^s) + \int\limits_{\mathcal{W}^s(\mathcal{M})} \int\limits_{\underline{w}^w}^{\kappa(w^s)} \phi'(R) dF^w(w^w) dF^s(w^s) = 1$$

Finally, and in order to obtain the same expression as the first-best, use the fact that the marginal utility of consumption is equal to one. Hence, the marginal rate of substitution is only expressed as $\phi'(R)$.

F.2 Optimal Marginal Tax Rate

To simplify the FOC with respect to τ, several results and assumptions are used. First, $g = {}^{(G_{V^i} \times V^i_C)}/_{\delta}$ is defined as the social valuation of agent's consumption. Second, $U^w(\kappa(w^s), R) = V^s(w^s, \mathcal{M})$, implying that some terms are canceling among them. Third, by assumption $\kappa(\hat{w}) = \overline{w}^w$, causing some integrals to collapse to zero. Fourth, as a result of the linearity in tax and the audit scheme, $\kappa'_\tau = 0$ for $w^s \leq w^*$ and for $w^s \geq \hat{w}$, resulting in some terms being equal to zero. Finally, the derivatives of consumption with respect to marginal tax in each sector are.

$$\frac{dC_w}{d\tau} = -w^w, \qquad \frac{dC_s}{d\tau} = \begin{cases} -w^s & \text{if } w^s < w^* \\ -w^* & \text{if } w^* \leq w^s \end{cases}$$

Using those facts and doing algebra yields

$$\int\limits_{\mathcal{W}^w(\mathcal{M})} g(w^w)(-w^w)dF^w(w^w)dF^s(w^s) + \int\limits_{\underline{w}^s}^{w^*}\int\limits_{\underline{w}^w}^{\kappa(w^s)} g(w^s)(-w^s)dF^w(w^w)dF^s(w^s) + \int\limits_{w^*}^{\hat{w}}\int\limits_{\underline{w}^w}^{\kappa(w^s)} g(w^s)(-w^*)dF^w(w^w)dF^s(w^s)$$
$$+ \int\limits_{\hat{w}}^{\overline{w}^s}\int\limits_{\underline{w}^w}^{\kappa(w^s)} g(w^s)(-w^*)dF^w(w^w)dF^s(w^s) = -\int\limits_{\mathcal{W}^w(\mathcal{M})} w^w dF^w(w^w)dF^s(w^s) + \tau\int\limits_{w^*}^{\hat{w}} \kappa'_\tau\kappa(w^s)f^w(\kappa(w^s))dF^s(w^s)$$
$$-\int\limits_{\underline{w}^s}^{w^*} w^s F^w(\kappa(w^s))\, dF^s(w^s) - \int\limits_{w^*}^{\hat{w}} w^* F^w(\kappa(w^s))\, dF^s(w^s) - \tau\int\limits_{w^*}^{\hat{w}} w^*\kappa'_\tau f^w(\kappa(w^s))\, dF^s(w^s) - \int\limits_{\hat{w}}^{\overline{w}^s} w^* F^w(\kappa(w^s))\, dF^s(w^s)$$

Let us note that $\kappa(w^s) - w^* = \dfrac{w^s - w^*}{1-\tau}$. By replacing it in the above equation and doing algebra, the optimal marginal tax formula is

$$\frac{\tau}{1-\tau}\int\limits_{w^*}^{\hat{w}}(w^s - w^*)\kappa_\tau(w^s)f^w(\kappa(w^s))dF^s(w^s) = \int\limits_{\mathcal{W}^w(\mathcal{M})}(1-g(w^w))w^w dF^w(w^w)dF^s(w^s) + \int\limits_{\underline{w}^s}^{w^*}\int\limits_{\underline{w}^w}^{\kappa(w^s)}(1-g(w^s))w^s dF^w(w^w)dF^s(w^s)$$
$$+\int\limits_{w^*}^{\hat{w}}\int\limits_{\underline{w}^w}^{\kappa(w^s)}(1-g(w^s))w^* dF^w(w^w)dF^s(w^s) + \int\limits_{\hat{w}}^{\overline{w}^s}\int\limits_{\underline{w}^w}^{\kappa(w^s)}(1-g(w^s))w^* dF^w(w^w)dF^s(w^s)$$

F.3 Optimal Budget for the IRS

Some results and assumptions are used to simplify the FOC with respect to B. First, the social valuation of the agent's consumption can be defined as $g = {}^{(G'_V \times V_C)}/_{\delta}$. Second, by definition $U^w(\kappa(w^s), R) = V^s(w^s, \mathcal{M})$, making some terms cancel among them. Third, $\kappa'_B = 0$ where $w^s \leq w^*$ and $w^s > \hat{w}$, resulting in some terms being equal to zero. Fourth, by definition $\kappa(\hat{w}) = w^w$, collapsing some terms to zero. Finally, the derivative of consumption in the self-employed sector with respect to the budget is

$$\frac{dC_s}{dB} = \begin{cases} 0 & \text{if } w^s < w^* \\ -\tau\dfrac{dw^*}{dB} & \text{if } w^* \leq w^s \end{cases}$$

Let us recall that, in the wage-earner sector, an increase in the budget for the IRS does not affect taxes. Using those facts and doing algebra yields

$$\tau \int_{w^*}^{\hat{w}} (w^* - \kappa(w^s)) \frac{dw^*}{dB} \kappa'_B f^w(\kappa(w^s)) dF^s(w^s) + \tau \int_{w^*}^{\hat{w}} \int_{\underline{w}^w}^{\kappa(w^s)} \frac{dw^*}{dB} dF^w(w^w) dF^s(w^s) + \tau \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^w}^{\kappa(w^s)} \frac{dw^*}{dB} dF^d(w^w) dF^s(w^s)$$

$$= 1 + \int_{w^*}^{\hat{w}} \int_{\underline{w}^w}^{\kappa(w^s)} g(w^s) \tau \frac{dw^*}{dB} dF^w(w^w) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^w}^{\kappa(w^s)} g(w^s) \tau \frac{dw^*}{dB} dF^w(w^w) dF^s(w^s)$$

G Differential Taxation

The Lagrangian of the government problem is

$$\begin{aligned} \mathcal{L} = & \int_{\mathcal{W}^w(\mathcal{M})} G\left(U^w(w^w, R)\right) dF^w(w^w) dF^s(w^s) + \int_{\mathcal{W}^s(\mathcal{M})} G\left(V^s(w^s, \mathcal{M})\right) dF^w(w^w) dF^s(w^s) \\ & - \delta \Bigg\{ B + R - t_w \int_{\mathcal{W}^w(\mathcal{M})} w^w dF^w(w^w) dF^s(w^s) - t_s \int_{\underline{w}^s}^{w^*} w^s F^w\left(\kappa(w^s)\right) dF^s(w^s) - t_s w^* \int_{w^*}^{\hat{w}} F^w\left(\kappa(w^s)\right) dF^s(w^s) \\ & - t_s w^* \int_{\hat{w}}^{\overline{w}^s} F^w\left(\kappa(w^s)\right) dF^s(w^s) \Bigg\} \end{aligned}$$

For simplicity, the notation over the Lagrange multiplier for the budget constraint is maintained. To simplify the derivative with respect to t_w, some assumptions and results are used. First, the social value of the agent's consumption is defined as $g = {}^{(G' \times U^w_C)}/_{\delta}$. Second, by definition $U^w(\kappa(w^s), R) = V^s(w^s, \mathcal{M})$, making some terms cancel among them. Third, by definition $\kappa(\hat{w}) = \overline{w}^w$, collapsing some terms to zero. Fourth, from the result in the IRS problem, $\kappa'_{t_w} = {}^{\kappa(w^s)}/_{(1 - t_w)}$ for $w^s \leq w^*$, $\kappa'_{t_w} = -\dfrac{t_s w^{*\prime}_{t_w}}{1 - t_w} + {}^{\kappa(w^s)}/_{(1 - t_w)}$ for $w^* < w^s \leq \hat{w}$, and $\kappa'_{t_w} = 0$ for $w^s > \hat{w}$. Fifth, ${}^{dC_w}/_{dt_w} = -w^d$. Sixth, ${}^{dC_s}/_{dt_d} = 0$ for $w^s \leq w^*$, and ${}^{dC_s}/_{dt_w} = -\dfrac{dw^*}{dt_w} t_s$ for $w^* < w^s \leq \hat{w}$. Using those facts and doing algebra yields

$$
\begin{aligned}
t_w = & \frac{\overbrace{\left(\int\limits_{\mathcal{W}^w(\mathcal{M})} (1-g(w^w))w^w dF^w(w^w)dF^s(w^s)\right)}^{A}(1-t_w)}{\underbrace{\int\limits_{\underline{w}^s}^{w^*} \kappa(w^s)^2 f^w(\kappa(w^s))dF^s(w^s) + \int\limits_{w^*}^{\hat{w}} \left(\kappa(w^s) - t_s w^{*\prime}_{t_w}\right)\kappa(w^s) f^w(\kappa(w^s))dF^s(w^s)}_{B}} \\
& + \frac{\overbrace{w^{*\prime}_{t_w}\left(\int\limits_{w^*}^{\hat{w}}\int\limits_{\underline{w}^w}^{\kappa(w^s)} (1-g(w^s))dF^w(w^w)dF^s(w^s) + \int\limits_{\hat{w}}^{\overline{w}^s}\int\limits_{\underline{w}^w}^{\kappa(w^s)} (1-g(w^s))dF^w(w^w)dF^s(w^s)\right)}^{D}(1-t_w)t_s}{\underbrace{\int\limits_{\underline{w}^s}^{w^*} \kappa(w^s)^2 f^w(\kappa(w^s))dF^s(w^s) + \int\limits_{w^*}^{\hat{w}} \left(\kappa(w^s) - t_s w^{*\prime}_{t_w}\right)\kappa(w^s) f^w(\kappa(w^s))dF^s(w^s)}_{B}} \\
& + \frac{t_s\overbrace{\left(\int\limits_{\underline{w}^s}^{w^*} w^s\kappa(w^s) f^w(\kappa(w^s))dF^s(w^s) + \int\limits_{w^*}^{\hat{w}} w^*\left(\kappa(w^s) - t_s w^{*\prime}_{t_w}\right) f^w(\kappa(w^s))dF^s(w^s)\right)}^{C}}{\underbrace{\int\limits_{\underline{w}^s}^{w^*} \kappa(w^s)^2 f^w(\kappa(w^s))dF^s(w^s) + \int\limits_{w^*}^{\hat{w}} \left(\kappa(w^s) - t_s w^{*\prime}_{t_w}\right)\kappa(w^s) f^w(\kappa(w^s))dF^s(w^s)}_{B}}
\end{aligned}
$$

Some parts are denoted with letters to simplify the exposure. The optimal tax rate is

$$
t_w = \frac{A + t_s D}{A + B + t_s D} + t_s \frac{C}{A + B + t_s D}
$$

In order to fulfill the limited liability constraint, the marginal tax rate in the wage-earner sector cannot be higher than one. Thus, it is necessary that $t_s \leq B/C$.

Now, the tax rate in the self-employed sector will be obtained. The same assumptions and results used to simplify the above derivation are also used now. Using those facts and doing algebra yields

$$t_s = \frac{\overbrace{\left[\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^w}^{\kappa(w^s)} (1-g(w^s))w^s dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^w}^{\kappa(w^s)} (1-g(w^s))(w^* + t_s w^{*\prime}_{t_s})dF^w(w^w)dF^s(w^s)\right]}^{E}(1-t_w)}{\underbrace{\int_{\underline{w}^s}^{w^*}(w^s)^2 f^w(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} w^*(w^* + t_s w^{*\prime}_{t_s})f^w(\kappa(w^s))dF^s(w^s)}_{F}}$$

$$+ \frac{t_w \overbrace{\left(\int_{\underline{w}^s}^{w^*}\kappa(w^s)w^s f^w(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}}\kappa(w^s)(w^* + t_s w^{*\prime}_{t_s})f^w(\kappa(w^s))dF^s(w^s)\right)}^{C}}{\underbrace{\int_{\underline{w}^s}^{w^*}(w^s)^2 f^w(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} w^*(w^* + t_s w^{*\prime}_{t_s})f^w(\kappa(w^s))dF^s(w^s)}_{F}}$$

Again, some parts are denoted with letters to simplify the exposure of the result. Note that the term C is equal to one in the derivation of t_w. Rearranging terms

$$t_s = \frac{E}{F} - t_w\frac{(E-C)}{F}$$

In this case, $t_s \leq 1$ is needed to fulfill the requirement for a direct incentive-compatible mechanism. Applying this condition to the above equation yields

$$\frac{E-F}{E-C} \leq t_w \leq 1 \Rightarrow F \geq C$$

Let us analyse if each possible combination of the marginal tax rates holds the condition for an optimum marginal tax rate obtained above. First, if $t_s < t_w$, $\kappa(w^s) > w^s$ for $w^s \leq \hat{w}$, hence $C > F$ produces a contradiction with the condition. Therefore, $t_s < t_w$ is not a solution. Second, if $t_s = t_w$, $\kappa(w^s) = w^s$ for $w^s \leq w^*$ and $\kappa(w^s) > w^s$ for $w^* \leq w^s \leq \hat{w}$, resulting in $C > F$. Therefore, $t_s = t_w$ cannot be a solution for differential taxation either. Third, if $t_s > t_w$, $\kappa(w^s) < w^s$ for $w^s \leq w^*$ and $\kappa(w^s) \lesseqgtr w^s$ for $w^* \leq w^s \leq \hat{w}$, resulting in $F \geq C$ for some cases. Hence, the only possible solution for differential taxation is $t_s > t_w$.

H Alternative Derivation of the Optimal Government Policies

In this appendix, the solution for the case where $\overline{n}^w > \overline{n}^s$ is shown. In each case, it is demonstrated that it is possible to extrapolate the conclusion given in the main section. For this explanation, let us define the level of productivity in the wage-earner sector that is equal to the threshold function evaluated in the upper bound of the self-employed productivity as $\hat{w} = \kappa(\overline{w}^s)$. The Lagrange of the government problem is

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G\left(U^d(w^d,R)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G\left(U^d(w^d,R)\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G\left(U^w(w^w,R)\right) dF^w(w^w)dF^s(w^s)$$

$$+ \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^w} G\left(U^w(w^w,R)\right) dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^w}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^w}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^w(w^w)dF^s(w^s)$$

$$- \delta \Bigg\{ B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^w dF^w(w^w)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^w} w^w dF^w(w^w)dF^s(w^s) - \tau\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^w dF^w(w^w)dF^s(w^s)$$

$$-\tau\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^w} w^w dF^w(w^w)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^w\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^*\int_{w^*}^{\overline{w}^s} F^w\left(\kappa(w^s)\right) dF^s(w^s) \Bigg\}$$

H.1 Optimal BLS Rule

The optimal BLS rule is obtained from the government problem deriving the Lagrangian with respect to the public goods. The first-order condition (FOC) with respect to R is

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} \frac{G'\left(U^w(w^w,R)\right)}{\delta}\phi'(R)dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^w} \frac{G'\left(U^w(w^w,R)\right)}{\delta}\phi'(R)dF^w(w^w)dF^s(w^s)$$

$$+ \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} \frac{G'\left(U^w(w^w,R)\right)}{\delta}\phi'(R)dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^w} \frac{G'\left(U^w(w^w,R)\right)}{\delta}\phi'(R)dF^w(w^w)dF^s(w^s)$$

$$+ \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^w}^{\kappa(w^s)} \frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^w}^{\kappa(w^s)} \frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^w(w^w)dF^s(w^s) = 1$$

The derivatives of the Lagrangian regarding consumption in each sector are

$$G'(U^w(w^w,R))f^w(w^w)f^s(w^s) - \delta f^w(w^w)f^s(w^s) = 0$$
$$G'\left(V^s(w^s,\mathcal{M})\right)f^w(w^w)f^s(w^s) - \delta f^w(w^w)f^s(w^s) = 0$$

where the definition of consumption, $C(w) = w - T(w)$, is used to replace taxes for consumption.

Using the above equation to replace the Lagrange multiplier in the FOC of the public good yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} \phi'(R)dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^w} \phi'(R)dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} \phi'(R)dF^w(w^w)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^w} \phi'(R)dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^w}^{\kappa(w^s)} \phi'(R)dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^w}^{\kappa(w^s)} \phi'(R)dF^w(w^w)dF^s(w^s) = 1$$

Finally, one can use the fact that the marginal utility of consumption is equal to one. Hence, the marginal rate of substitution is only $\phi'(R)$. The final expression is

$$\phi\prime(R) = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} MRS(RC)dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^w} MRS(RC)dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} MRS(RC)dF^w(w^w)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^w} MRS(RC)dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^w}^{\kappa(w^s)} MRS(RC)dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^w}^{\kappa(w^s)} MRS(RC)dF^w(w^w)dF^s(w^s) = 1$$

This equation is the same as in the paper, only the limits in the integrals change, but it represents the same result: the solution of the optimal public good provision is the same as in the first-best.

H.2 Optimal Marginal Tax Rate

Let us use several results and assumptions from the model to simplify the first-order condition with respect to τ. First, $g = (G_{V^i} \times V_C^i)/\delta$ can be defined as the social valuation of the agent's consumption. Second, $U^w(\kappa(w^s), R) = V^s(w^s, \mathcal{M})$, implying that some terms are canceling among them. Third, from the IRS solution, $\kappa'_\tau = 0$ for $w^s \leq w^*$. Fourth, the derivatives of consumption with regard to marginal tax in each sector are

$$\frac{dC_w}{d\tau} = -w^w, \qquad \frac{dC_s}{d\tau} = \begin{cases} -w^s & \text{if } w^s < w^* \\ -w^* & \text{if } w^* \leq w^s \end{cases}$$

Fifth, $\kappa(w^s) - w^* = \dfrac{w^s - w^*}{1-\tau}$. Using this fact and doing algebra, the optimal marginal tax formula is

$$\frac{\tau}{1-\tau}\int_{w^*}^{\overline{w}^s} [(w^s - w^*)\kappa_\tau(w^s)f^w(\kappa(w^s))]\, dF^s(w^s) = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} (1-g(w^w))w^w dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^W} (1-g(w^w))w^w dF^w(w^w)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} (1-g(w^w))w^w dF^w(w^w)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^w} (1-g(w^w))w^w dF^w(w^w)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^w}^{\kappa(w^s)} (1-g(w^s))w^s dF^w(w^w)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\underline{w}^w}^{\kappa(w^s)} (1-g(w^s))w^* dF^w(w^w)dF^s(w^s)$$

This formula is essentially the same as in the main section, only the limits of some integrals change.

H.3 Optimal Budget for the IRS

Let us use some results and assumptions to simplify the first-order condition with respect to B. First, the social valuation of the agent's consumption is defined as $g = {G'_V \times V_C}/{\delta}$. Second, by definition $U^w(\kappa(w^s), R) = V^s(w^s, \mathcal{M})$, causing some terms to cancel among them. Finally, the derivative of consumption in the self-employment sector with respect to the budget is

$$\frac{dC_s}{dB} = \begin{cases} 0 & \text{if } w^s < w^* \\ -\tau \dfrac{dw^*}{dB} & \text{if } w^* \leq w^s \end{cases}$$

At this point, it is important to recall that, in the wage-earner sector, an increase in the budget of the IRS does not affect taxes. Using those facts and doing algebra yields

$$\tau \int_{w^*}^{\overline{w}^s} (w^* - \kappa(w^s))\kappa'_B f^w(\kappa(w^s))dF^s(w^s) + \tau \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^w}^{\kappa(w^s)} w^*_B dF^w(w^w)dF^s(w^s) = 1 + \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^w}^{\kappa(w^s)} g(w^s)\tau w^*_B dF^w(w^w)dF^s(w^s)$$

This equation represents the same as the one shown in the main section.