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Abstract: The paper provides a framework to approach price and quantity determination in the roundwood market from a slightly new perspective. In the spirit of the trade union literature, a model of timber price determination is formulated according to which the forest owners' association determines timber prices and then forest firms unilaterally decide upon the timber to be used. The novelty here is to incorporate investment decisions of firms as a strategic factor into the model. The structure of the model is the following. The game is played in two stages. First, the forest owners' association and the firms in the forest industry decide on timber prices and capital stock, respectively. In the second stage, the firms determine timber demand conditional on the timber price-capital stock game, so that the equilibrium concept is the subgame perfect Nash equilibrium. This game-theoretic model is applied to the annual data from the Finnish paper and pulp industry over the period 1960-1992. Estimation and testing results concerning the price and quantity determination of timber as well as capital stock determination are generally favorable for the hypotheses presented. In particular, diagnostics and various test procedures indicate that these equations outperform conventional specifications derived from the theory of the demand for factors of production.

Keywords: timber prices, forest sector, Nash equilibrium
JEL-classification: L73, J51, Q23

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Tiivistelmä: Kirjoituksessa tarkastellaan uudesta näkökulmasta raakapuun hinnan ja määrän määräytymistä raakapuumarkkinoilla. Ns. ammattiliittojen teorian mukaisesti muotoillaan raakapuumarkkinoiden hinnanmääräytymismalli, jossa metsänomistajien yhdistys asettaa raakapuun hinnan, jonka jälkeen metsäteollisuuden yritykset päättävät puun kysynnästä. Malliin on liitetty myös yritysten investointipäätökset strategisena muuttujana ja sen ajallinen rakenne on seuraava. Metsänomistajien yhdistyksen ja yritysten välinen peli on kaksivaiheinen. Ensimmäisessä vaiheessa metsänomistajien yhdistys ja yritykset päättävät kantohinnoista ja pääomakannasta. Toisessa vaiheessa yritykset päättävät puun kysynnästä annettuna kantohinnat ja pääomakanta. Kyseessä on siis alipelitäydellinen Nash tasapaino. Tätä peliteoreettista mallia sovelletaan Suomen paperiteollisuutta koskevaan vuosiaineistoon ajanjaksolle 1960-1992. Estimointi- ja testaustulokset tukevat pääsääntöisesti mallia. Diagnostiikan ja erilaisten testimenettelyjen perusteella peliteoreettisesta mallista johdetut puun hintayhtälö, investointien määräytymisyhtälö ja puun ehdollinen kysyntäyhtälö ovat parempia kuin tavanomaiset täsmennykset.

Avainsanat: kantohinnat, metsäsektori, Nash tasapaino
JEL-luokitus: L73, J51, Q23.

1. INTRODUCTION

Traditionally, price and quantity determination in the roundwood market has been approached by using the assumption of competitive roundwood markets. The equilibrium price is determined by assuming that the demand for and the supply of timber are equal. This makes it possible to estimate demand and supply functions as well as the determinants of equilibrium prices in terms of exogenous demand and supply parameters. The demand for timber is postulated to result from the profit-maximizing behavior of firms as an application of the theory of the demand for factors of production. As for the supply of timber, it is often described by using a simple two-period intertemporal model, in which the forest owners decide upon the harvesting, i.e. how much to harvest now and in the future. [1]

Price and quantity determination in the roundwood market have been empirically studied in the above-mentioned framework, e.g., in Johansson and Löfgren (1985, Ch. 9), in Hultkrantz and Aronsson (1989), using Swedish data, and in Hetemäki and Kuuluvainen (1992), using Finnish data. Though the performance of the estimating equations has usually been relatively good in many respects, these models, however, suffer from some weaknesses. First, the results are not always very sensible. For instance it is not very easy to accept the result that timber is a substitute for labor.[2] Second, the competitive model does not directly account for the possibility that investments in the forest industry affect roundwood markets and, vice versa, that the price of roundwood and its expected development may affect capital stock decisions of firms in the forest industry. These interactions may be potentially important. Finally, and importantly, one can criticize the assumption that prices are determined so as to equalize demand and supply. A better hypothesis might be that prices are subject to negotiations between firms and forest owners. For instance in Finland and Sweden both forest owners and forest industry are well-organized, and negotiations on timber prices have been an everyday practice since the 1960s.

The purpose of this paper is to provide a framework which approaches price and quantity determination from this slightly new perspective. In the spirit of the trade union literature we formulate the model of timber price determination in accordance with the notion that the forest owners' association determines timber prices, after which firms decide upon timber used. The novelty here is to incorporate investment decisions of firms into the model. The stress on investment decision reflects the fact that firms in the modern paper and pulp industry are big and

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[1] See e.g. Koskela (1989) and Ollikainen (1993) for applications of two-period harvesting models to various issues of forest taxation.

[2] See e.g. Hetemäki and Kuuluvainen (1992).

capital intensive. Both parties of negotiation know well that new investments will have effects on the demand for timber and quite likely take this into account in their strategic behavior.[3]

The game is assumed to be played in two stages. First, the forest owners' association and firms decide on timber prices and capital stock, respectively. In the second stage, firms determine the demand for timber conditional on timber price and capital stock. Thus the equilibrium concept is the subgame perfect Nash equilibrium. Finally, we apply the resulting game-theoretic model of the determination of prices and quantities of timber and capital stock to the annual data of the Finnish paper and pulp industry over the period 1960-1992. We compare the resulting specifications concerning prices and quantities of timber and capital stock to the alternative, somewhat more traditional hypotheses. All in all, the empirical results are generally quite favorable for the hypotheses implied by the game theoretic model, thus suggesting that the approach is promising. In particular, diagnostics and various test procedures indicate that both the timber price equation and the conditional demand for timber equation outperform specifications implied by the conventional theory of the demand for factors of production.

The paper is organized as follows: In section 2 the model is presented, the equilibrium is defined and various comparative static properties are developed. Section 3 is devoted to estimation results and to various testing procedures for the annual data from the Finnish paper and pulp industry over the period 1960-1992. Finally, there is a brief conclusion.[4]

2. THEORETICAL MODEL: A TWO-STAGE GAME APPROACH

The model to be developed in this section tries to capture the crucial features of the annual timber price negotiation practice in Finland. The prevailing system can be described as follows. The representatives of forest industry and forest owners (Central Union of Agricultural Producers and Forest Owners (MTK)) started to make voluntary timber price recommendations during the 1960s. The first real timber price agreement was signed for the cutting year 1978-79. During the 1980s both parties supplemented the timber price agreement by a voluntary recommendation recarding the timber quantity to be sold. Since then the negotiation system has worked until today. As the evidence of timber price negotiations is so obvious, it is worthwhile to examine seriously at how

3 There is little research about the functioning of the roundwood market under imperfect competition. Notable exceptions are the studies by Johansson and Löfgren. They modeled roundwood markets as monopsonistic market, where the local industry gets a spatial monopsony due to transportation costs offers, and they also analyzed the price determination subject to bargaining between forest owners and forest industry, when forest owners association maximizes utility of harvest revenue over the competitive timber price level and forest industry maximizes its profits (see Johansson and Löfgren 1985, Ch. 8). They, however, neither allowed for the capital stock determination nor evaluated the bargaining hypothesis empirically.

4 Some of the technical material as well as the description of the data have been presented in Koskela and Ollikainen (1996) and are available from the authors upon request.

this hypothesis works empirically and to test it against alternative hypothesis of roundwood market. In this section we formulate a model for timber price determination in accordance with the notion that the forest owners' association determines the timber price, while firms in forest industry decide upon the amount of timber used.[5]

2.1 Structure of the Model

The firms in the forest industry are assumed to produce the final product (e.g. pulp) by using two inputs, namely capital (k) and roundwood (x). To keep the analysis as simple and clear as possible we assume that the production function is of the Cobb-Douglas form, with constant returns to scale according to (1).

$$(1) \qquad Q = f(k,x) = k^a x^{1-a}, \quad 0 < a < 1$$

The firms face a downward-sloping demand curve given in equation (2).

$$(2) \qquad ZD(p) = Zp^{-\varepsilon}$$

The demand curve is given in a separable form (see e.g. Nickell (1978)), where the parameter Z describes the position of the demand curve. Moreover, it is isoelastic with elasticity of demand being given by the parameter ε. The firms produce what is demanded so that $Q = D(p)Z$. This makes it possible to express the price as a function of the quantity supplied, $p = p(zQ)$, where $z = Z^{-1}$. With the functional forms (1) and (2) one obtains $p = (zk^a x^{1-a})^{-1/\varepsilon}$. As a special case we have the situation of perfect competition when the price of the final product is exogenously given from the viewpoint of an individual firm.

The profit maximization problem of the firms in forest industry can be expressed in a general form as maximizing

$$(3) \qquad \pi = p[zf(k,x)]f(k,x) - rk - qx,$$

[5] Here we draw from the literature on trade unions. See Anderson and Devereux (1988) and Holm, Honkapohja and Koskela (1994) in particular. One may criticize the assumption that the forest owners' union sets the timber price. A more realistic assumption might be that the timber price is determined in a bargain between the forest owners' union and firms. This 'right-to-manage' approach does, however, yield qualitative results similar to those achieved when the forest owners' union unilaterally determines timber price.

with respect to x and k, where p is the price of final output, r the real interest rate and q the price of timber.

Forest owners supply roundwood to the market provided that timber price is high enough. We assume that they are represented by a forest owner's association. The forest owners' association chooses a timber price so that the product of timber times real timber price over a given reservation price is maximized.

(4) $$U = \left[\frac{q}{p} - e(r)\right]x,$$

where $e(r)$ is the reservation price of timber, which is assumed to depend negatively on the real interest rate r, i.e., $e'(r) < 0$.[6] According to the formulation (4) the forest owners' association is not willing to sell timber if the real timber price falls below the reservation price.

The next task is to solve the Nash equilibrium of the model in the standard way of backward induction, starting with the last stage of the game and utilizing this solution in solving the first stage.

2.2 Nash Equilibrium

In the second stage, the firms in the forest industry decide how much timber to use, given the optimal choice of k and q during the first stage. This amounts to maximizing (3) by choosing x so as to maximize their profits. At the interior solution firms maximize their profits by equating the marginal revenue of timber use to its factor price as in (5).

(5) $$MR(p)f_x = q,$$

where $MR(p) = p + p'z$.

The second-order condition is given by equation (6).

[6] It is easy to show by using an intertemporal model of harvesting behavior that the willingness to sell timber - the reservation price - depends negatively on the real interest rate. (A proof is available from the authors upon request.) Reservation price could also depend on the price in the international roundwood markets to which forest owners supply timber if they don't get higher price in the domestic markets. Alternatively, e could reflect forest owners' trade-off between harvest income and valuation of amenities of forests meaning that they are not willing to give up their possibilities of enjoying the amenities unless they get the price which exceeds e. In what follows we do not develop these alternatives.

(6) $\pi_{xx} = MR(p)f_{xx} + MR'(p)f_x z < 0$,

where $MR'(p) = p' + p''z$. Nonincreasing marginal revenue is a sufficient, but not necessary condition for this to hold.

Utilizing the specified functional forms (1) and (2), we solve explicitly solve for the optimal demand for timber in terms of exogenous variables.[7] Derived demand for timber is given in a qualitative form in equation (7). A rise in the capital stock (other input), a fall in the exogenous component of demand $z = Z^{-1}$ (via the inverse demand $p = p(zQ)$) and a rise in the price of final product p all boost the demand for timber, whereas a higher real timber price decreases it.

(7) $x^* = x^*(\underset{+}{k}, \underset{-}{q}, \underset{-}{z}, \underset{+}{p})$

We now move on to the first stage of the game, in which the firms in the forest industry and the forest owners' association decide on the capital stock and timber price, respectively, taking the other player's decision variable as given. Using (7) as a constraint in these optimizations provides the requirement for the subgame perfection of the equilibrium.

The firms in the forest industry decide the optimal capital stock by choosing k so as to maximize profits (8) taking (7) as a constraint and the timber price q as given.

(8) $\pi = p\left[zf\left(k, x^*(k,q,p,z)\right)\right]f\left(k, x^*(k,q,p,z)\right) - rk - qx^*(k,q,p,z)$

This gives, using (5) as the envelope condition, equation (9), which has similar interpretation as earlier: the marginal revenue of capital use must be equal to its factor price at $\pi_k = 0$.

(9) $MR(p)f_k = r$

[7] Using (1) and (2) makes it possible to write the demand for timber as $x^* = \alpha k p^{\eta} q^{-\eta}$, where $\alpha \equiv (1-a)^{\eta}(1-1/\varepsilon)^{\eta} > 0$ and $\eta = -\dfrac{qx_q}{x} = \dfrac{1}{a} > 1$ denotes the price elasticity of timber demand.

Substituting this for x in $p = (zk^a x^{1-a})^{-\frac{1}{\varepsilon}}$ and rearranging yields $p = c(zk)^{-\frac{\psi}{\eta-1}} q^{\psi}$, where $c = \alpha^{-\psi/\eta}$ and $\psi = \dfrac{\eta-1}{\eta+\varepsilon-1}$, so that $0 < \psi < 1$. One should thus notice that changes in z, k and q also change the price of the final good, which must be taken into account when solving for the comparative statics of z, k and q.

The second-order condition is given in

(10) $\pi_{kk} = MR(p)f_{kk} + MR'(p)zf_k < 0$

and it can be shown to hold under the specified functional forms (1) and (2).

The first-order condition (9) gives the optimal capital stock for a given timber price and is thus *the reaction function of firms in the forest industry*. Using the specified functional forms (1) and (2), the first-order condition (9) can be solved explicitly for the capital stock.[8] The partial equilibrium capital stock depends positively on the price of the final output and on the amount of timber x^*, and negatively on the real interest rate as well as on the exogenous inverse component of demand z due to its effect on the price of final good (see footnote 8). Thus we can write the capital stock equation qualitatively as

(11) $k^* = k^*(\underset{+}{p}, \underset{-}{r}, \underset{-}{z}, \underset{+}{x^*})$.

The forest owners' association optimizes (4) subject to (7), taking the capital stock exogenously given. This leads to the following first-order condition

(12) $U_q = \frac{1}{p} - \frac{\eta}{q}\left(\frac{q}{p} - e(r)\right) = 0$,

where the association is assumed to account for the endogeneity of the price of final good p, and η is the price elasticity of timber demand. According to (12), the timber price is set so as to equate the marginal benefit from increasing timber price ($\frac{1}{p} > 0$) to the marginal cost due to a resulting fall in timber demand when q increases ($-\frac{\eta}{q}\left(\frac{p}{q} - e(r)\right) < 0$).

The second-order condition is given in

(13) $U_{qq} = -\frac{(\eta - 1)}{pq}(1 - \psi) < 0$

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[8] This yields the following expression for k: $k^* = nx^* p^\beta r^{-\beta}$, where $n = ((1 - 1/\varepsilon)a)^\beta$ and $\beta = (1 - a)^{-1}$.

and it holds, as $\eta > 1$ and $\psi < 1$.

Equation (12) gives the optimal timber price for a given capital stock and thus defines *the reaction function of the forest owners' association.* It can be written as

(14) $q^* = pe\left(\frac{\eta}{\eta - 1}\right)$, where $e = e(r)$.

Thus the partial equilibrium timber price depends positively on the reservation and output prices and the mark-up of real timber price over the reservation price $\left(\frac{\eta}{\eta - 1}\right)$. Instead the negative demand shift and the real interest rate affect timber price negatively. Thus we can write qualitatively

(15) $q^* = q^*(\underset{+}{e}, \underset{+}{p}, \underset{-}{z}, \underset{-}{r})$,

where the negative demand shift affects via the price of final product *p*, (i.e. $p_z < 0$, see footnote 8).

The optimal choices of the firms in the forest industry and of the forest owners' association in equations (9) and (12) respectively define the Nash equilibrium of the capital stock - timber price game. They define the reaction functions of forest industry, *H*, and forest owners' association, *G*, i.e., give the best choice of *k* or *q*, given the other's choice of *q* or *k*.

(16)
a) $H(k, x, q; r, z, e) \equiv \pi_k = 0$
b) $G(k, x, q; r, z, e) \equiv U_q = 0$

Due to the second-order conditions for the choice of capital stock and timber price, one obtains for reaction functions that $H_k = \pi_{kk} < 0$ and $G_q = U_{qq} < 0$. To determine the slopes of the reaction functions, one has still to analyze the signs of the cross derivatives $H_q = \pi_{kq}$ and $G_k = U_{qk}$. As $H_q < 0$ and $G_k < 0$ both the industry's and the forest owners' association's reaction function is downward-sloping in the (*q*,*K*) space.[9] Finally, for the Nash equilibrium to be stable, it is required

[9] It can be shown that $H_q = \pi_{kq} = \frac{r}{q}\left(\frac{(\eta-1)(1-\varepsilon)}{\eta+\varepsilon-1}\right) < 0$ and $G_k = U_{qk} = -\frac{(1-\eta)}{pk}\left(\frac{1}{\eta+\varepsilon-1}\right) < 0$.

(in addition to the second-order conditions for the maximization of the target functions for firms and the forest owners' association) that the determinant of the matrix of the second-order derivatives must be positive, so that $\pi_{kk}U_{qq} - \pi_{kq}U_{qk} > 0$. As we have $\pi_{kq}U_{qk} > 0$, the stability of the Nash equilibrium presupposes that the reaction function H must be steeper than the reaction function G.[10] Optimal capital stock and timber price (k^*, q^*) is determined at the point where the reaction functions intersect in the (*q*,*k*) space in Figure 1, which describes a stable Nash Equilibrium.

2.3 Comparative Statics of Capital Stock and Timber Price

The final step in the analysis of the model is the derivation of comparative static properties of the Nash equilibrium. This requires the determination of the dependence of *H* and *G* on the exogenous variables *r, z* and *e*. For the reaction function of the firms in the forest industry one obtains

(17) $H_r < 0$, $H_z < 0$ and $H_e = 0$.

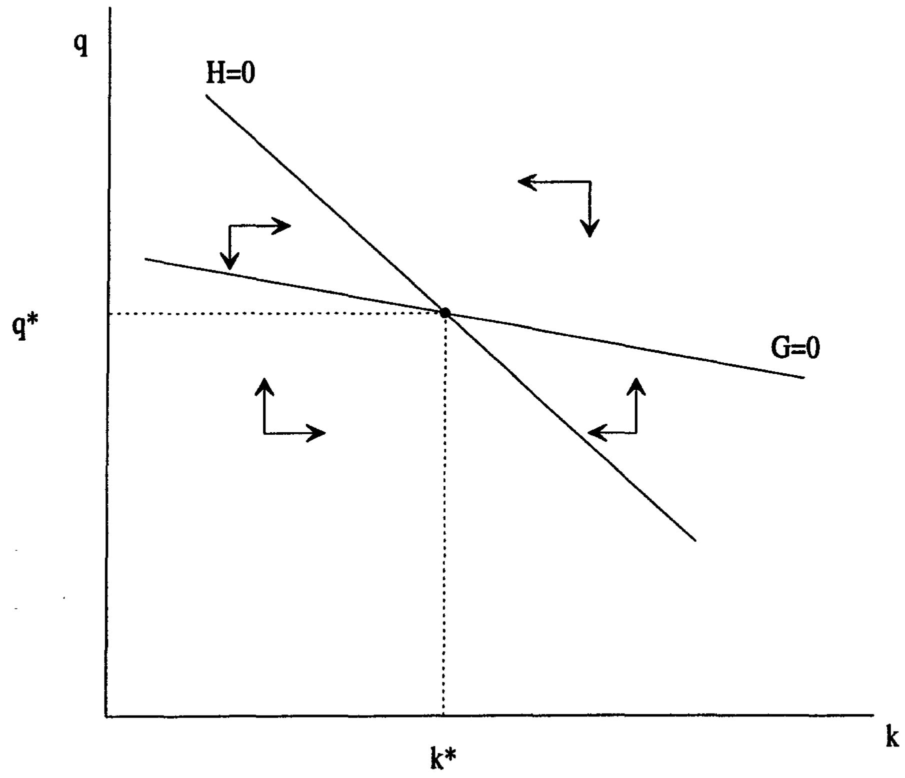
The effects of exogenous variables on the forest owners' association reaction function are respectively

(18) $G_r < 0$, $G_z < 0$ and $G_e > 0$.

The shifts of the reaction functions condensed in equations (17) and (18) can be combined to offer a convenient graphical presentation of the comparative statics of the Nash equilibrium in Figure 2. The original Nash equilibrium is given by the intersection of the reaction functions (solid lines). The dashed lines in the diagram represent the shifts of the reaction functions due to changes in exogenous variables. These changes produce various new equilibria, which are marked by capital letters from A to C. Table 1 shows the qualitative results of the comparative statics. The first column in Table 1 tells which exogenous variable is changing, the second indicates its effect on the reaction functions, the third identifies the new equilibrium point in Figure 2 and the fourth column gives the effects of exogenous variables on the capital stock, timber price and demand for timber, respectively.

[10] Under specifications (1) and (2), this condition can be shown to hold at the interior solution. For further details of stability analysis in the oligopoly context, see Dixit (1986).

Figure 1: Stable Nash Equilibrium



According to the comparative statics, a rise in the real interest rate shifts both reaction functions to the left and we have three possible outcomes: both k and q decrease (A'), q increases and k decreases (A) or q decreases and k increases (A"). It is easy to show that the effect on the demand for timber is ambiguous (A'), negative (A) or positive (A"), respectively. A rise in reservation price of timber decreases capital stock and increases timber price. Via both channels the demand for timber decreases. The effect of a rise in the (inverse) demand shift parameter for final products has a negative effect on both reaction functions. Again we have three possible outcomes: both q and k decrease (A'), q increases and k decreases (A) or q decreases and k increases (A"). In all cases, however, the effect on the demand for timber remains ambiguous.

Figure 2: Comparative Statics of the Nash equilibrium

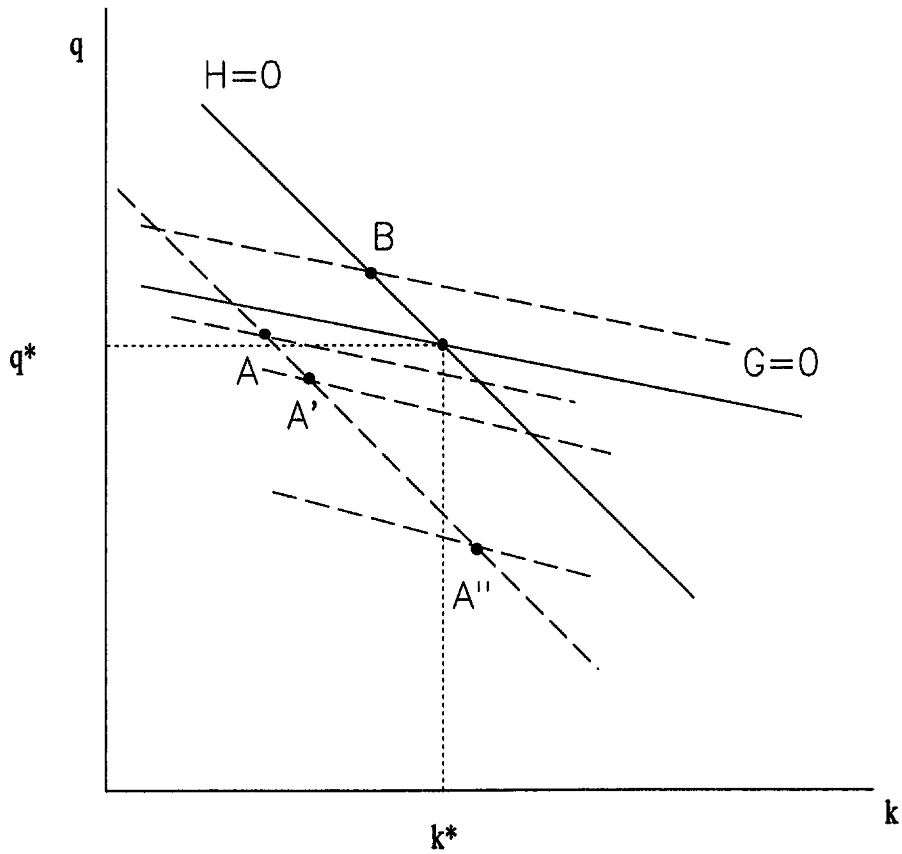


Table 1: Comparative Statics of the Nash Equilibrium

Type of a shift in exogenous variable	Effect on reaction function	New eq. point at	Effect on capital, timber price and demand for timber
$\Delta r > 0$	$H_r < 0 \quad G_r = 0$	A	$\Delta k < 0,\ \ \Delta q > 0,\ \ \Delta x < 0$
		A'	$\Delta k < 0,\ \ \Delta q < 0,\ \ \Delta x = ?$
		A"	$\Delta k > 0,\ \ \Delta q < 0,\ \ \Delta x > 0$
$\Delta e > 0$	$H_e = 0 \quad G_e > 0$	B	$\Delta k < 0,\ \ \Delta q > 0,\ \ \Delta x < 0$
$\Delta z > 0$	$H_z < 0,\ G_z < 0$	A	$\Delta k < 0,\ \ \Delta q > 0,\ \ \Delta x = ?$
		A'	$\Delta k < 0 \ \ \Delta q < 0,\ \ \Delta x = ?$
		A"	$\Delta k > 0 \ \ \Delta q < 0,\ \ \Delta x = ?$

3. EMPIRICAL APPLICATION TO THE FINNISH PAPER AND PULP INDUSTRY

We now move on to the empirical part of the paper. We start by postulating dynamics associated with the capital stock, timber price and timber demand equations. Then we specify the corresponding equilibrium equations and combine them with the dynamics to be estimated. Finally, the FIML estimation results for the system of capital stock - timber price and the OLS estimation results for timber demand are presented for the Finnish paper and pulp industry over the period 1960-1992. In this context we test for the structure of decisions implied by the two-stage game, and we contrast our timber demand equation -- conditional on timber prices and capital stock -- with the determination of the quantity of timber suggested by the conventional theory of the demand for factors of production.

3.1 Dynamics and specification of equations

According to the time structure of the model, timber price q and capital stock k are determined simultaneously, and conditional on these, timber demand x is determined recursively. Hence, it is appropriate to discuss the dynamics of adjustment separately for timber price and capital stock on the one hand and for timber demand on the other.

For the adjustment process for timber price and capital stock, we follow a formulation now commonly used in the stability analysis of oligopoly equilibria (for details, see Dixit 1986). According to this myopic rule, forest owners and firms in paper and pulp industry increase timber price and capital stock, respectively, when they perceive positive marginal utility and positive marginal profit from doing so. Using this standard oligopoly stability analysis and taking the linear approximation around the equilibrium *(logk*, log q*)*, the following dynamic system is obtained

(19a) $$D\log k = \lambda_{11}\left[\log(k_{-1}) - \log(k^*)\right] + \lambda_{12}\left[\log(q_{-1}) - \log(q^*)\right]$$

(19b) $$D\log q = \lambda_{21}\left[\log(k_{-1}) - \log(k^*)\right] + \lambda_{22}\left[\log(q_{-1}) - \log(q^*)\right],$$

where D is the difference operator and where we have used the log-formulation for the Nash equilibrium. For the system (19a)-(19b) the stability conditions require that the eigenvalues of the

(20) $$I + \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = I + \Lambda$$

are inside the unit circle (here I is the unit matrix).

As for the determination of timber demand we do not postulate any dynamics, but assume that it adjusts within a year to various shocks perturbing the equilibrium conditional timber demand. The theoretical framework in section 2 suggests potentially relevant variables and their a priori signs in the determination of the equilibrium values for capital stock, timber prices and timber demand. Assuming the log-linear functional form we obtain the following equilibrium system of equations

$$\textbf{(21a)}\quad \log k^* = \alpha_1 + \underset{(-)}{\alpha_2} \log\left[\frac{q^*}{p}\right] + \underset{(-)}{\alpha_3} r + \underset{(?)}{\alpha_4} \log Z$$

$$\textbf{(21b)}\quad \log q^* = \beta_1 + \underset{(+)}{\beta_2} \log p + \underset{(?)}{\beta_3} \log Z + \underset{(+)}{\beta_4} \log k^*$$

$$\textbf{(21c)}\quad \log x^* = \gamma_1 + \underset{(-)}{\gamma_2} \log\left[\frac{q^*}{p}\right] + \underset{(+)}{\gamma_3} \log k^* + \underset{(?)}{\gamma_4} \log Z$$

where the variables with asterisk refer to equilibrium values and the variables in parentheses refer to hypothesized signs. In what follows, we do not test for the restrictions implied by the specified functional forms (1) and (2) utilized in the derivation of comparative static results so that the theoretical exercise should thus be seen here as suggesting qualitative hypotheses. Lack of appropriate data is the reason why the equation (21b) does not include the reservation price *e* of the forest owners' association.

According to equation 21a (21b), the firms' (the forest owners') actions depend on the forest owners' (the firms') reaction function. This kind of interdependence is problematic in the sense that if equations (21a) and (21b) describe the simultaneous move game, then the capital stock and timber price equations cannot be identified in the dynamic system (21a-21b). In order to avoid this identification problem, the variables log(p*) and log(k*) are replaced by forecasts $\log(q^f)$ and $\log(k^f)$, which are made at time $t-1$ for the Nash equilibrium prevailing at time t.[11]

[11] The forecasts for $\log(q^f)$ were obtained by modeling timber prices as an AR-process. The capital stock forecasts ($\log(k^f)$) were obtained as follows. According to the capital depreciation formula, capital stock forecast is a function of investment forecast I^f such that $k^f - k_{-1} = I^f - \delta k_{-1}$, where δ is the capital depreciation coefficient. Firm's investment plans made in previous spring were used as a proxy for I^f. For an evaluation of δ, see Koskela and Ollikainen (1996).

Equations (21a-21c) are estimated for the Finnish paper and pulp industry using annual data over the period 1960-1992. Data sources and construction of some variables are explained in Koskela and Ollikainen (1996). The basic set of dynamic equations (19a-19b) jointly with the equilibrium equations (21a-21c) provide the dynamic equations to be estimated. For convenience these are expressed below.

(22a) $$\log k = \alpha_1 + \alpha_2 \log(k_{-1}) + \alpha_3 \log\left[\frac{q}{p}\right] + \alpha_4 r + \alpha_5\left[\log(q_{-1}) - \log(q^f)\right] + \alpha_6 \log Z + \varepsilon$$

(22b) $$\log q = \beta_1 + \beta_2 \log(q_{-1}) + \beta_3 \log p + \beta_4 \log k + \beta_5\left[\log(k_{-1}) - \log(k^f)\right] + \beta_6 \log Z + \omega$$

(22c) $$\log x^* = \gamma_1 + \gamma_2 \log\left[\frac{q}{p}\right] + \gamma_3 \log kk + \gamma_4 \log Z + \upsilon$$

where ε, ω and υ refer to the error terms of the equations. Note that q in capital stock equation (4a) and k in nominal timber price equation (4b) are endogenous, while q and k in timber demand equation (9) are exogenous according to the hypotheses given in section 2. After some experimentation we decided to substitute log *kk* - the capital stock adjusted by its utilization rate - for log *k* in (22c).

According to the time structure of the model, timber prices and capital stock are determined simultaneously, and timber demand is determined recursively conditional on the timber price-capital stock game. This structure was tested by carrying out Hausman's exogeneity test for the system (22a-22c) on an equation-by-equation basis (for details, see e.g. Davidson & MacKinnon 1993, 237-242). Contrary to our theoretical reasoning, we could not reject at the 1% significance level the hypothesis that the capital stock is an exogenous variable in the demand for timber equation, but the hypothesis that timber price is an exogenous variable in the capital stock equation was rejected. Despite this slightly mixed evidence we proceeded to system estimation of capital stock and timber price.[12]

The FIML (full information maximum likelihood) estimation results of the system of equations (22b) and (22c) together with some diagnostics are presented in Table 2. The estimation results can be briefly summarized as follows:[13]

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[12] A complete set of results from the exogeneity tests is available from the authors upon request.
[13] The system of equations was also estimated by 2SLS method. Results turned out to be very similar to those of FIML. Hence they are not reported here, but are available from the authors upon request.

Table 2: FIML Estimation Results of the Capital Stock-Timber Price System

variable	*log k*	*log q*
Constant	3.86 (4.00)	- 18.88 (-3.10)
$\log(k_{-1})$	0.59 (5.12)	
log (q/p)	- 0.13 (-4.55)	
r	0.30 (2.54)	- 2.19 (-4.28)
$\log(q_{-1}) - \log(q^f)$	- 0.11 (-1.95)	
log Z	0.17 (3.78)	- 1.06 (-3.96)
$\log(q_{-1})$		0.10 (1.16)
log p		0.80 (6.55)
log k		2.18 (3.35)
$\log(\delta k_{-1}) - \log(I^f)$		- 0.25 (-4.33)
Diagnostics for	single equations	
χ^{2v}_{nd}	1.33 (0.52)	0.86 (0.65)
F_{ar}	F(2,19) = 4.66 (0.02)	F(2,19) = 0.21 (0.73)
F_{arch}	F(1,19) = 0.15 (0.70)	F(1,19) = 0.01 (0.91)
F_{het}	F(15,5) = 0.34 (0.95)	F(15,5) = 0.58 (0.81)
Diagnostics for	the whole system	
	χ^{2v}_{nd} 3.27 (0.51) F^{v}_{ar} F(8,36)=1.63 (0.35)	

χ^{2}_{nd} refers to the Jarque-Bera normality test statistics, F_{ar} to the second-order autocorrelation test statistics, and F_{arch} and F_{het} describe the ARCH and WHITE heteroscedasticity test statistics, respectively. Finally χ^{2v}_{nd} is a test for normality for the whole system (see Doornik and Hansen (1994)) and F^{v}_{ar} is a test for the vector autocorrelation. The numbers in parenthesis for parameter estimates are t-values and those for the diagnostics are the marginal significance levels (see Doornik and Hendry (1991) for details).

First we look at the stability of Nash equilibrium associated with the dynamic structure of the system (19a-b). Because the system has been estimated in the level form, we have to check whether the eigenvalues of $I + \Lambda$ are inside the unit circle. The eigenvalues of $I + \Lambda$ from Table 2 are given after calculation in

$$\textbf{(23)} \qquad \cong \frac{.69 \pm \sqrt{.35}}{2},$$

which are clearly less than one in absolute value. Hence, the capital stock-timber price system is stable and the adjustment coefficients lie in conformity with the oligopoly stability requirements.

Second, the diagnostic performance of the system of equations is good. The single equation evaluation statistics for no heteroscedasticity (F_{arch} against first-order heteroscedasticity and F_{het}) and of no non-normality (χ^{2v}_{nd}) show good performance. Instead there is some slight evidence for serial autocorrelation (F_{ar} against second-order autocorrelation) at the 5 per cent but not 1 per cent significance level in the case of the capital stock equation. But as for the system (vector) tests, neither normality nor lack of autocorrelation can be rejected at the standard significance levels.

Third, the parameter estimates are rather precisely estimated and of expected sign from the viewpoint of our theoretical reasoning. The capital stock equation adjusts sluggishly; the real timber price (log(*q*/*p*)) affects it negatively and the variable describing the position of the demand curve (log Z) positively. Moreover, a rise in the forecast made at time $t-1$ for the Nash equilibrium q^* prevailing at time *t*, which was calculated by using an AR-forecast, decreases the capital stock, although the parameter estimate is small and statistically significant at the 6% level. Finally, the real interest rate has a positive coefficient estimate, which is significant at the 2% level. This lies in conformity with the theoretical model; while the partial equilibrium effect of the real interest rate on capital stock is negative, the 'total' effect can be of any sign. As for the timber price equation, the timber price adjusts sluggishly, and is positively and significantly affected by the price of the final good. The investment plans which are included into the term $\log(\delta k_{-1})-\log(I^f)$ will have a significantly positive effect on the timber price. This is natural; when firms plan to invest more, forest owners tend to increase timber price. The demand and capital stock variables are statistically significant at the 1% level. The effect of the former is negative while that of the latter is positive. Both coefficient estimates lie in conformity with the hypothesis presented earlier.[14] Finally and importantly, the real interest rate affects timber price negatively and statistically very significantly.

3.2 Testing for alternative specifications of the demand for timber

Next, we move on to look at the specification of the demand for timber, which according to our hypothesis is determined recursively conditional on the timber price-capital stock Nash equilibrium. We contrast this specification (model A in what follows) with two alternative hypotheses. According to one hypothesis the demand for timber is affected by the relative prices of factors of

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14 In footnote 8 (p. 7) we presented an explicit form for the equation of the price of the final good. In logarithmic form it is given by $\log(p) = const + \beta_0(\log k - \log z) + \beta_1 \log q + u$, where u is an error term. OLS estimation results supported this specification. The coefficient restriction for $(\log k - \log z)$ could not be rejected, coefficient estimates were of expected sign ($0 > \beta_0 > -1$ and $0 < \beta_1 < 1$) and highly significant. This gives further support for the approach. (A complete set of results is available from the authors upon request.)

production as well as by the output. This cost-minimization hypothesis is presented as specification B. According to another hypothesis the demand for timber is affected by its real price and the real interest rate as well as the variable describing the position of the demand curve in the output market. This imperfect competition hypothesis -- according to which firms face a downward-sloping demand curve -- is presented as specification C. Specifications B and C can be derived from the theory of the demand for factors of production, when firms in the paper and pulp industry face the excess supply of goods (model B) or the downward-sloping demand in the goods market (model C), respectively.

The OLS estimation results are presented in Table 3, where we have used the lagged real timber price in the model specifications C and A in order to capture overlapping cutting periods. In what follows we start by discussing the properties of our proposed specification A. Estimation results can be briefly characterized as follows. First, the performance of the equation is relatively good; there are no signs of misspecifications in terms of the diagnostics of the error term. Normality, autocorrelation, heteroscedasticity and functional form test statistics indicate that the error term does not suffer from these diagnostic problems. Second, with the exception of the demand variable, which can be of any sign according to our theoretical model, the explanatory variables are significant and behave according to the hypothesis suggested by the theoretical model. The capital stock adjusted by its utilization rate has a positive, while the (lagged) real timber price a negative effect on the demand for timber, and both coefficient estimates are statistically significant at the standard levels.

We also made some further checks. First, we tested for the parameter constancy of model specification A by replacing a standard assumption that there is a single structural break in the sample by a more general one stating that the parameters of the model may change continuously over time.[15] The idea was to present the smooth transition regression model (STR) as an alternative to the parameter constancy model, in which the transition from "one regime to another" was determined by the *hth* degree polynomial of the time trend. There is no economic theory available for choosing h so that we used a statistical selection technique based on a short sequence of nested tests as in Granger and Teräsvirta (1993, Ch.7). The test statistics for values $h = 1,2,3$ can be called *LM1*, *LM2* and *LM3*, respectively.[16] The following F-tests were obtained for the specification A: *LM1*: $F(4,24) = 3.38$ (0.025); *LM2:* $F(8,20) = 1.83$ (0.129) and *LM3:* $F(12,16) = 2.09$ (0.085), where the numbers in parenthesis are marginal significance levels. Thus, only the simplest *LM1*-test rejected the null hypothesis of parameters constancy at the 5%, but not at the 1% significance level.

15 See Lin and Teräsvirta (1994) for how this can be done.

16 If h=1, there is only one break and the change is monotonic. If h=2, the change can be symmetric around the break point, while for h=3, one can have several breaks and the change need not be monotonic. See Lin and Teräsvirta (1994) for details.

Hence, these analyses give support to the notion that the conditional demand for timber displays parameter stability.

Second, we compared the conditional demand for timber with two alternative specifications B and C (Table 3). These are dominated by our proposed hypothesis in terms of the overall performance of the equations; the goodness-of-fit statistics and diagnostics of the error term show worse performance for B and C than for A. The goodness-of-fit is significantly worse for B, and normality of the error term can be rejected at the 1% significance level. For the specifation C, there are signs of heteroscedasticity and the functional form misspecification.[17]

Table 3: OLS Estimation Results of Alternative Timber Demand Equations

variable	***model A***	***model B***	***model C***
constant	-11.13 (-4.72)	-1.76 (-0.79)	-1.63 (-1.68)
r		1.41 (1.46)	1.23 (1.57)
$\log(q/p)$-1	-0.79 (-4.70)		-0.84 (-4.05)
$\log Q$		0.25 (1.90)	
$\log kk$	1.16 (4.34)		
$\log Z$	-0.21 (-1.68)		0.27 (3.06)
Diagnostics			
R^2	0.62	0.14	0.41
χ^2_{nd}	0.83 (0.66)	8.56 (0.01)	1.53 (0.46)
DW	1.74	1.27	1.15
F_{ar}	F(2,26) = 0.91 (0.42	F(2,27) = 1.87 (0.17)	F(2,27) = 2.95 (0.07)
F_{arch}	F(1,26) = 0.03 (0.87	F(1,27) = 0.25 (0.62)	F(1,26) = 0.01 (0.93)
F_{het}	F(6,21) = 0.90 (0.5[illegible]	F(5,23) = 0.46 (0.80)	F(9,18) = 2.43 (0.05)
F_{reset}	F(1,27) = 2.29 (0.1[illegible]	F(1,28) = 0.03 (0.86)	F(1,27) = 11.07 (0.00)

R^2 is the goodness-of-fit statistic, χ^2_{nd} is the normality test statistics, DW is the Durbin-Watson statistic for the first-order autocorrelation, F_{ar} is the test statistic for the second-order autocorrelation, F_{arch} and F_{het} are heteroscedasticity test statistics, and F_{reset} describes the test statistics for the functional form. Numbers in parentheses for the coefficient estimates are t-values, while those for the diagnostics are marginal significance levels.

17 Finally, we made some further comparisons by doing two types of non-nested tests, namely the J-test suggested by Davidson and MacKinnon (1981) and the encompassing test suggested by Mizon and Richard (1986). According to these tests, our proposed specification A performed best. For a complete set of results, see Koskela and Ollikainen (1996).

4. CONCLUDING REMARKS

The purpose of the paper has been to provide a framework which approaches price and quantity determination in the roundwood market from a slightly new perspective. In the spirit of the trade union literature, a model of timber price determination was formulated in which the forest owners' association determines timber prices after which forest firms unilaterally decide upon timber used. The game was played in two stages. First, the forest owners' association and the firms in forest industry decide on timber prices and capital stock, respectively. In the second stage, the firms determine timber demand conditional on the timber price-capital stock game, so that the equilibrium concept was the subgame perfect Nash equilibrium. The model was applied to the annual data from the Finnish paper and pulp industry over the period 1960-1992. Estimation and testing results concerning the price and quantity determination of timber as well as capital stock determination turned out to be generally favorable for our hypotheses.

More specifically, the FIML estimation results of the capital stock-timber price system can be briefly summarized as follows. Its diagnostic performance is good, and the parameter estimates are rather precisely estimated and are of the expected sign. As for the capital stock equation, the demand variable has a significantly positive, while the real interest rate and real timber price have a negative effect. Timber price is positively and significantly affected by the price of the final good, by the investment plans of the firms in the paper and pulp industry, and by the capital stock. Estimation results for the equation of the demand for timber suggests that the capital stock adjusted by its utilization rate has a positive and significant, while the (lagged) real timber price has a negative and significant effect on the demand for timber. Finally, the conditional demand for timber specification, as proposed by our game-theoretic model, dominates some common alternative specifications both in terms of diagnostics and various test procedures.

REFERENCES:

Anderson, S. & Devereux, M. (1988): "Trade Unions and the Choice of the Capital Stock", the *Scandinavian Journal of Economics*, 27-44.

Aronson, T. (1990): The Incidence of Forest Taxation - A Study of the Swedish Roundwood Market. *the Scandinavian Journal of Economics*, 92, 65-79.

Davidson, R. & MacKinnon, J.G. (1981): "Several Tests for Model Specification in the Presence of Alternative Hypotheses", *Econometrica*, 781-793.

Davidson, R. & MacKinnon, J.G. (1993): *Estimation and Inference in Econometrics.* Oxford University Press, New York.

Dixit, A. (1986): "Comparative Statics for Oligopoly", *International Economic Review*, 107-122.

Doornik, J.A. and Hansen, H. (1994): "A practical test for univariate and multivariate normality" *Discussion Paper, Nuffield College.*

Doornik, J.A. and Hendry, D.F. (1991): *PcFiml 80: Interactive Econometric Modelling of Dynamic Systems*, London, International Thomson Publishing.

Granger, C.W.J. and Teräsvirta, T. (1993): *Modelling Nonlinear Economic Relationships*, Oxford University Press, Oxford.

Hetemäki, L. & Kuuluvainen, J. (1992): "Incorporating Data and Theory in Roundwood Supply and Demand Estimation", *American Journal of Agricultural Economics*, 1010-1018.

Hultkrantz, L. & Aronsson, T. (1989): "Factors Affecting the Supply and Demand for Timber from Private Nonindustrial Lands in Sweden: An Econometric Study", *Forest Science,* 946-961.

Holm, P. & Honkapohja, S. & Koskela, E. (1994): "A Monopoly Union Model of Wage Determination with Capital and Taxes: An Empirical Application to the Finnish Manufacturing", *European Economic Review*, 285-303.

Johansson, P.-O. & Löfgren, K.-G. (1985): *The Economics of Forestry & Natural Resources*, Basil Blackwell, Oxford.

Koskela, E. (1989): "Forest Taxation and Timber Supply under Price Uncertainty and Perfect Capital Markets", *Forest Science*, 137-159.

Koskela, E. and Ollikainen, M. (1996): A Game-Theoretic Model of Timber prices with Capital Stock: An Empirical Application to the Finnish Forest industry. Mimeo.

Lin, C-F.J. & Teräsvirta T. (1994): "Testing the Constancy of Regression Parameters Against Continuous Structural Change", *Journal of Econometrics*, 62, 211-228.

Mizon, G.E. & Richard, J.-F. (1986): "The Encompassing Principle and Its Application to Testing Nonnested Hypotheses", *Econometrica*, 657-677.

Nickell, S.J. (1978): *The Investment Decisions of Firms*, Cambridge University Press, Cambridge.

Ollikainen, M. (1993) : "A Mean-Variance Approach to Short-Term Timber Selling and Forest Taxation Under Multiple Sources of Uncertainty", *Canadian Journal of Forest Research*, 23, 573-581.

Appendix 1 : Nash equilibrium and comparative statics

This appendix presents all the results given in the paper with Cobb-Douglas production function and separable, isoelastic demand.

The conditional demand for timber: (The second stage of the game)

(1) $$\underset{\{x\}}{Max}\ \pi = (zk^a x^{1-a})^{-1/\varepsilon}(k^a x^{1-a}) - rk - qx,$$

where $p = \left(zk^a x^{1-a}\right)^{-1/\varepsilon}$ is the separable, isoelastic demand.

The first-order condition defines the optimal conditional demand for timber

(2) $$\pi_x = \frac{1-a}{x}(k^a x^{1-a})p\left(1-\frac{1}{\varepsilon}\right) - q = 0 \quad \Rightarrow x^* = \alpha k p^\eta q^{-\eta}$$

where $\eta \equiv -\frac{qx_q}{x} = \frac{1}{a} > 1$ is the price elasticity of timber demand and $\alpha \equiv (1-a)^\eta (1-\frac{1}{\varepsilon})^\eta > 0, \varepsilon > 1$.

Optimal capital stock and timber price: Nash equilibrium (The second stage of the game)

In the first stage of the game firms decide on k and the forest owners' association on q given the conditional demand for timber $x^* = \alpha k p^\eta q^{-\eta}$.

(3)

a) $$\underset{\{k\}}{Max}\ \pi = (zk^a x(k,q,p,z)^{1-a})^{-\frac{1}{\varepsilon}} k^a x(k,q,p,z)^{1-a} - rk - qx$$

b) $$\underset{\{q\}}{Max}\ U = \left(\frac{q}{p} - e(r)\right)x$$

The first-order conditions define optimal k and q as follows

(4)

a) $$\pi_k = \frac{a}{k}(k^a x^{1-a})p(1-\frac{1}{\varepsilon}) - r = 0 \qquad \Rightarrow k^* = nxp^\beta r^{-\beta}$$

b) $$U_q = \frac{1}{p} - \frac{\eta}{p}\left(\frac{q}{p} - e(r)\right) = 0 \qquad \Rightarrow q^* = pe(r)\left(\frac{\eta}{\eta - 1}\right)$$

where $n = \left((1-\frac{1}{\varepsilon})a\right)^\beta$ and $\beta \equiv (1-a)^{-1}$.

The second-order conditions are

(5)

$$\text{a)}\ \pi_{kk} = -\frac{r}{k}\left[-(1-\frac{1}{\eta}) - \frac{1}{\eta+\varepsilon-1}\right] < 0$$

$$\text{b)}\ U_{qq} = -\frac{(\eta-1)}{pq}(1-\psi) < 0$$

Moreover, we have $\pi_{kq} = \frac{r}{q}(\frac{(\eta-1)(1-\varepsilon)}{\eta+\varepsilon-1} < 0$ and $U_{qk} = \frac{(1-\eta)}{pk}\frac{1}{\eta+\varepsilon-1} < 0$

The stability of Nash equilibrium requires that

(6) $$\Delta = \begin{vmatrix} \pi_{kk} & \pi_{kq} \\ U_{qk} & U_{qq} \end{vmatrix} > 0,$$

which holds as $\Delta = \frac{(\eta-1)r}{pqk(\eta+\varepsilon-1)}[\varepsilon\eta + (\eta-1)(\varepsilon+\eta-1)] > 0$.

Comparative statics of the Nash equilibrium can be calculated as follows

(7) $$(\Delta)\begin{bmatrix} \pi_{kk} & k_\theta^* \\ U_{qk} & q_\theta^* \end{bmatrix} = \begin{bmatrix} -\pi_{k\theta} \\ -U_{q\theta} \end{bmatrix},$$

where θ refers to exogenous variables.

Total comparative statics of the demand for timber

The change in the demand for timber due to changes in exogenous variables is obtained as follows.

$$\text{a)}\ \frac{dx}{dr} = x_q q_r + x_k k_r < 0$$

(8) $$\text{b)}\ \frac{dx}{de} = x_q q_e + x_k k_e < 0$$

$$\text{d)}\ \frac{dx}{dz} = x_z + x_q q_z + x_k k_z = ?$$

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