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TESTING NONLINEAR DYNAMICS, LONG MEMORY AND CHAOTIC BEHAVIOUR WITH FINANCIAL AND NONFINANCIAL DATA***

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Abstract

This paper contains a set of tests for nonlinearities in economic time series. The tests comprise both standard diagnostic tests for revealing nonlinearities and some new developments in modelling nonlinearities. The latter test procedures make use of models in chaos theory, so-called long-memory models and some asymmetric adjustment models. Empirical tests are carried out with Finnish monthly data for twelve macroeconomic time series covering the period 1920-1996. Test results support unambiguously the notion that there are strong nonlinearities in the data. The evidence for chaos, however, is weak or nonexisting. The evidence on long memory (in terms of so-called rescaled range and fractional differencing) is somewhat stronger although not very compelling. Nonlinearities are detected not only in a univariate setting but also in some preliminary investigations dealing with a multivariate case. Certain differences seem to exist between nominal and real variables in nonlinear behaviour.

Tiivistelmä

Tässä tutkimuksessa testataan taloudellisiin aikasarjoihin liittyviä epälineaarisuuksia. Testit koostuvat sekä tavanomaisista diagnostisista testeistä että eräistä uusista epälineaarisuuden olemassaoloa selvittävistä testimenetelmistä. Jälkimmäiset testit liittyvät kaaosteorian sovellutuksiin, ns. Pitkän muistin malleihin ja epäsymmetrisen sopeutumisen malleihin. Empiiriset analyysit tehdään 12 Suomea koskevalla aikasarjalla, jotka kattavat kuukausitasolla ajanjakson 1920-1996. Testit tukevat kiistatta sitä oletusta, että aikasarjoissa on epälineaarisuuksia. Epälineaarisuudet eivät kuitenkaan välttämättä heijasta determinististä kaaosta. Aikasarjojen pitkää muistia koskeva evidenssi (joka perustuu ns. uudelleenskaalatun vaihtelun (rescaled range) tarkasteluihin ja osittaisen differenssoinnin malleihin) on jonkin verran voimakkaampaa, mutta se ei ole mitenkään tilastoaineistoa dominoiva piirre. Edellä mainittuja ominaisuuksia ilmenee sekä yksittäisten muuttujien suhteen mutta myös tutkittaessa muuttujien välisiä riippuvuuksia. Nimellisten ja reaalisten aikasarjojen välillä näyttää olevan jonkin verran eroja epälineaarisuuksien määrässä ja luonteessa.

Contents

1 Introduction

Even though economic relationships are thought to be fundamentally nonlinear, most modelling practices start with linear tests and modelling. The obvious reason for this has been the difficulty of choosing from among numerous nonlinear alternatives. Economic theory rarely helps the researcher with anything other than perhaps giving the assumed sign between the two variables. Given the amount of tests and statistical theory based on linear spaces, it has been almost too easy to restrict attention to linear models. However, the poor performance of these models in forecasting e.g. business cycles suggests that maybe things are not so simple.

Apart from some almost self-evident nonlinear functions like the production function or the utility function, nonlinearities have rarely been treated satisfactorily in economics. Although the state of art in nonlinear economics has started to receive more attention, the main problem is that we do not have any clear-cut procedure for approaching these nonlinearities. Up till now, we have had no better advice than just to begin with linear testing and to try to limit the nature of nonlinearities to some well specified class of models.

This paper examines several long Finnish macroeconomic time series. The purpose of the examination is to find out whether there are, in fact, any signs of nonlinearities in these series. We carry out a set of tests analogously to Lee, White and Granger (1993). Most of these tests are applied to univariate models although a multivariate application would obviously be more interesting. When scrutinizing the series we pay special attention to the distinction between nominal and real series. This can be motivated by the fact that nonlinearities are presumably quite different depending on whether nominal or real variables are involved. (For an extensive survey of the literature, see Mullineux and Peng (1993).) Thus, it is of some interest to compare a typical real series, say industrial production, and a nominal series, say stock prices, in this respect.

Most monetary series - like relative prices, changes in price level and money aggregates - display some form of nonlinear behaviour. Prices are often more volatile than the real series, since they play a market-clearing role. Monetary phenomena are based upon valuations that can be adjusted without any significant cost. In the market-clearing situation it is often - but not necessarily always - easier to change the price rather than the quantity. Although prices can easily move in both directions, crises in the market produce excessively large negative (positive) changes. Nominal price rigidities would also have similar effects. Therefore it comes as no surprise that the real exchange rate, stock prices or inflation seem to adjust asymmetrically to shocks.

This affects the volatility of these series. Another major observation concerning the origin of "price shocks" relates to their unstable variance in time. It has been shown that in many cases price changes - e.g. in the stock market - cluster significantly. Forecasting price changes is therefore a harder task for economic agents than forecasting smoother real variables.

Nowadays, a general response to situations of changing volatility (heteroskedasticity) is to use an ARCH model specification. It may well be, however, that the ARCH model is not the proper framework. It is possible that prices possess the so-called long memory property, thus containing permanent components. In particular, the long-memory property shows up in high and persistent serial correlation over long lags between absolute values of the (linearly filtered) series. It also shows up in so-called rescaled range analysis, which provides estimates of the persistence of time series. Obviously, this kind of long-memory phenomenom is at variance with a linear structure and therefore it may be useful to consider it here as well.

However, in many cases real economic variables also vary in a nonlinear way. Evidence of nonlinear adjustment is provided by e.g. the apparent and persistent tendency for there to be cycles in most important production variables (see, e.g., Pfann and Palm (1993) for details). Whether these nonlinearities in real series arise from the generating process of a series itself or from random shocks is largely an empirical question. So far, no agreement has emerged as to whether real or monetary phenomena are responsible for business cycles. We hope that our estimates of the nonlinearity of these series might shed some light on this issue as well.

One general class of explanations for nonlinearities is chaotic behaviour. Quite recently, there have been numerous theoretical and empirical applications of "chaos theory". In particular, the behaviour of financial variables has been analyzed from this point of view (see, e.g., the books by DeGrauwe et al (1993), Greedy and Martin (1994), Peters (1993) and Vaga (1994)). The analyses have concentrated on testing the existence of chaos; theoretical analyses have mainly been presented as examples of various cases where (determinisitic) chaos might arise. Here, we leave the theoretical developments aside and concentrate solely on empirical testing. It is not easy to derive a theoretical model which would be readily applicable to all macroeconomic series which are at our disposal.

Although the analysis mainly deals with univariate models, some preliminary work is done to identify nonlinear relationships between variables. In this context, we do not follow any specific hypothesis concerning the relationships between variables. Rather, we simply make use of a cross-correlation analysis with respect to different moments of our variables. Thus, the analyses represent some sort of first step towards a generalized Granger test for nonlinear relationships. This analysis gives us a general idea of the magnitude and nature of these relationships. An obvious next step is to go back to theory and think about how the findings coincide with different theoretical approaches.

The structure of the paper is very straightforward. First, we look at the data in section 2, then we briefly present the test statistics and illustrate their properties with some simulated data in section 3, and in section 4 we go through the test results for univariate models. The results deal with various diagnostic tests procedures and with a set of analyses ot the correlation dimension, rescaled range, time irreversibility, nonlinear adjustment, parameter stability and long memory. In section 5, we scrutinize the results from a cross-correlation analysis between different moments of these series and, finally, in section 6 we present some concluding remarks.

2 The data

The data are monthly Finnish data covering the period 1920M1-1996M9. After data transformations the period 1922M5-1996M9 is covered. Thus, there are 893 observations in each series. The following twelve series are analyzed:

Industrial production (ip)
Bankruptcies (bank)
Terms of trade (tt)
Real exchange rate index (fx)
Yield on long-term government bonds (r)
Consumer price index (cpi)
Wholesale price index (wpi)
Banks' total credit supply (credit)
Narrow money (M1)
Broad money (M2)
UNITAS (Helsinki) stock exchange index (sx)
Turnover in stock exchange (st).

The first four series are real and the subsequent eight nominal. The data are presented in Figure 1. For presentational convenience, most of the series are shown in logs. To get some idea of the timing of changes in these variables the recession periods are marked by shaded areas.

Otherwise, the details of the data are presented in Virén (1992), Autio (1997) and Poutavaara (1996). We merely point out that the ip, bank, credit, M1 and M2 series are seasonally adjusted. This is simply for data reasons - only seasonally adjusted data were available for the prewar period 1920-1938. As for World War II (1939-1945), the data are treated in the same way as for the peace years.

The overall quality of the time series is rather good. Only the interest rate series are somewhat deficient, as can also be seen from Figure 1. The interest rate series suffers from the fact that banks' borrowing and lending rates were administratively fixed from the mid-1930s to the early 1980s and, therefore, bond yields were not genuinely market-based but were, too, indirectly rationed.

3 The test statistics

Data transformations

It is preferable to start testing nonlinearities by estimating the linear model and analyzing the respective residuals. Although economic relationships are more likely to be nonlinear, there is a danger of unnecessary complication if the difference in relation to a linear model is small.

The need for a nonlinear model also depends on the purpose of the model. For short-run forecasting, linear models may suffice, but for long-run forecasts or explanation of apparent nonlinear features more appropriate modelling is needed. Since testing linearity is widely covered in Granger and Teräsvirta (1993), we discuss only a few basic considerations here. The linearity tests can be divided into two groups, depending on whether a specific nonlinear alternative exists or not. Since our data do not refer to any specific nonlinear formulation, we concentrate on testing against a general nonlinear alternative.

As was mentioned above, we analyze only univariate models. Some kind of basic specification is provided by a linear AR(4), which turned out to be a reasonably good approximation for all time series. In specifying the order of the autoregressive models, we used model selection criteria (SC, HQ, AIC). In order to study the dynamic dependencies between variables, we thought that in the first place it would be best to filter the original series with the linear autoregressive model of the same order. Thus, the residuals are not severely (linearly) autocorrelated. A few exceptions do exist, however, for higher order autocorrelation (for the lag 12, for instance). Anyway, we prefer the parsimonious AR(4) model to more sophisticated specifications.[1] In fact, we also used first log differences for all relevant variables instead of AR(4) residuals. The residuals were qualitatively very similar, suggesting that the AR(4) transformation is not that crucial. For space reasons, the results with the first difference data are not reported here.

Log transformations were applied to most of the series. Thus, only the terms of trade, the real exchange rate and the interest rate series were left untransformed. To assess the validity of this transformation we made use of the Box-Cox transformation. The results of this procedure generally supported the above-mentioned choice. Only in the case of consumer prices and the terms of trade could one not be sure whether or not to make the log transformation.

Dealing with nonlinearities is often easier after the linear dependencies in a time series have already been taken care of. Therefore nonlinear adjustment can be found in a series properly filtered with an autoregressive (linear) model. However, empirical problems do emerge at this point. It often happens, especially in multivariate analysis, that filtering is almost too effective, since all the significant relationships between variables are removed. Therefore unduly long autoregressive lag models that also affect the asymmetricity in the series should be avoided.

Standard diagnostic tests

Given the autoregressive model, we compute the following sets of tests: First a set which consists of some basic statistics on the residuals of this linear AR(4) model (see Table 1). These statistics include the coefficients of skewness and kurtosis in addition to the median. We intend to use these data to detect possible asymmetries. The second set of tests consists of traditional specification tests for functional misspecification/nonlinearity. The tests (reported in Table 2) consist of Engle's (1982) ARCH test in terms of lagged squared residuals, Ramsey's (1969) RESET test in terms of higher-order powers of the forecast value of x_t, White's (1980) heteroskedasticity/functional form misspecification test in terms of all squares and

[1] We are well aware that the remaining higher-order autocorrelation might invalidate the subsequent test statistics which are related to the measure of the correlation dimension (see Ramsey, 1990, for details).

cross-products of the original regressors, the Jarque and Bera (1980) test for normality of residuals and Tsay's (1986) nonlinearity test in terms of squared and cross-products of lagged values x_t.[2] Finally, the Hsieh (1991) third-order moment coefficients are computed. They should detect models which are nonlinear in mean and hybrid models which are nonlinear in both mean and variance but not models which are nonlinear in variance only.

The BDS test for chaotic process

In addition to these "traditional" test statistics we also computed the BDS (Brock, Dechert and Scheinkman) test statistic (see Table 4) and Ramsey's (1990) irreversibility $G_{1,2}$ test. The BDS test is designed to evaluate hidden patterns of systematic forecastable nonstationarity in time series. The test was originally constructed to have high power against deterministic chaos, but it was discovered that it could be used to test other forms of nonlinearities as well (see, e.g., Brock, Hsieh and LeBaron (1991) Frank and Stengos (1988a) and Medio (1992) for details).

The BDS test can also be used as a test for adequacy of a specified forecasting model. This can be accomplished by calculating the BDS test for the standardized forecast errors. The BDS test is then used as a specification test. If no forecastable structure exists among forecast errors, the BDS test should not exceed the critical value. The BDS test has been found to be useful as a general test for detecting forecastable volatility. The key concept here is the correlation dimension, which can be applied in detecting the topological properties of series. For a purely random variable, the correlation dimension increases monotonically with the dimension of the space and the correlation dimension remains small even when the topological dimension of the space (embedding dimension) increases (Brock, Hsieh and LeBaron (1991)).

For a single series x_t for which $x_{t,m}$ is the set of m adjacent values of this time series $x_{t,m} = \{x_t, x_{t+1}, ..., x_{t+m-1}\}$, also called m-histories of x, the m-correlation integral $C_m(\epsilon)$ is defined as

$$C_m(\varepsilon) = \lim_{T\to\infty} T^{-2}[\text{number of ordered pairs } (x_{t,m}, x_{s,m}), t \neq s,\ 0<s,t<N,$$
$$\text{such that } \|x_{t,s} - x_{s,t}\| < \varepsilon],$$

where $T = N - m + 1$ and N is the length of the series. $\|x\|$ denotes the maximum norm (see, e.g. Eckmann and Ruelle (1985)). Now, defining the correlation dimension d(m) as

[2] For the properties of these test statistics see e.g. Petruccelli (1990) and Lee, White and Granger (1993).

$$d(m) = \lim_{\epsilon \to 0} \frac{\partial \log C_m(\epsilon)}{\partial \log \epsilon}$$

The correlation dimension is based on the fact that, for small ϵ, $C_m(\epsilon) \sim \epsilon^d$. In the case of truly chaotic series, the correlation dimension is independent of m while if the series are random i.i.e. processes $C_m(\epsilon) = C_1(\epsilon)^m$ and hence the (regression) slope of log(C) on log(ϵ) increases monotonicly with m.

The purpose of the correlation measure is to describe the complexity of the true series and measure the nonlinear dimension (degrees of freedom) of the process. Tests of chaos concentrate on low-dimensional deterministic chaos processes, since there is no efficient way to tell the difference between high-dimensional chaos and randomness.

Although the correlation dimension can be calculated and interpreted rather easily, there are some major problems with the estimation of this measure, mainly due to fact that economic data are relatively noisy and there are too few observations available (see Ramsey (1990) and Ramsey, Rothman and Sayers (1991) for more details). It can be shown that when the dimension of the data set is based on this Grossberger-Procaccia measure, the estimate of it is necessarily biased because of the following small sample problem: With a finite data set the value of ϵ cannot be too small because otherwise $C_m(\epsilon)$ will be zero and thus d(m) is not defined. By contrast, with large values of ϵ, $C_m(\epsilon)$ saturates at unity so that the regression of $\log(C_m)$ on log(ϵ) is simply zero. Thus, the smaller the number of observations, the larger ϵ has to be, and the more biased the estimate of the dimension will be.

Although theory concerns the properties of $C_m(\epsilon)$ as $\epsilon \to 0$, the reality is that the range of ϵ used in estimating d(m) is far from zero and inevitably increases away from zero as the embedding dimension is increased. Smaller values of ϵ require substantial increases in sample size in order to determine a linear relationship between $\log(C_m(\epsilon))$ and log(ϵ). In fact, the relationship is linear only for a narrow range of values for ϵ. Thus, one should be very careful in evaluating single point estimates of d(m). By scrutinizing the entire path of d(m) with respect to ϵ one may obtain a more reliable estimate of the true dimension. Alternatively, one may use the test procedure suggested by Brock, Hsieh and LeBaron (1991) in calculating the following BDS test statistic:

$$BDS(m,\epsilon) = \sqrt{T}(\hat{C}_m(\epsilon) - [\hat{C}_1(\epsilon)]^m)/\sigma(m,\epsilon),$$

where $\sigma(m,\varepsilon)$ is an estimate of the standard deviation. The BDS tests whether $C_m(\varepsilon)$ is significantly greater than $C_1(\varepsilon)^m$, and when this is the case nonlinearity is present. Under the null hypothesis of x_t following i.i.d., and for fixed m and ε, $C_{m,T}(\varepsilon) \to C(\varepsilon)^m$, as $T \to \infty$, and BDS(m,ε) has the standard normal distribution. (Notice, however, that $C_m(\varepsilon) = C(\varepsilon)^m$ does not imply i.i.d..) The power of the test will depend critically on the choice of ε.

The BDS test statistic is complicated since it depends on the embedding dimension (m) and the chosen distance (ε) related to the standard deviation of the data. The selection of m is important in small samples, especially when m is large, since increasing m means that the number of nonoverlapping sequences will become smaller. And when the sample is less than 500 the asymptotic distribution may be different from the sampling distribution of the BDS statistic. The selection of ε is even more crucial and failure to detect non-normality in calculating the BDS with small ε is a consequence of too few observations. Brock, Hsieh and LeBaron (1991, p. 52) suggest that for 500 or more observations, the embedding dimension m should be smaller or equal to 5, whereas ε should be 0.5–2 times the standard deviation of the data. In the empirical application, some alternative values of the dimension parameter m and the distance parameter ε are used.

The problem with the BDS test is, however, that it does not have a simple interpretation. Nonlinearity based on the BDS test could be a result of chaos or a nonlinear stochastic process. However, the BDS test was originally designed to test whether the data-generating process of a series is deterministic (chaotic) or not (Granger & Teräsvirta (1993), p. 63). Since the BDS test is based on the null hypothesis that the observations (here AR(4) residuals) are i.i.d., rejection merely reveals that this is not the case. The specific form of nonlinearity is therefore an open question.

As for the practical implementation of the test, it is done here by using the residuals of the AR(4) model as inputs. The use of the autoregressive filter is based on the invariance property of chaotic equations shown by Brock (1986). Brock showed that if one carries out a linear transformation of chaotic data, then both the original and the transformed data should have the same correlation dimension and the same Lyapunov exponents.

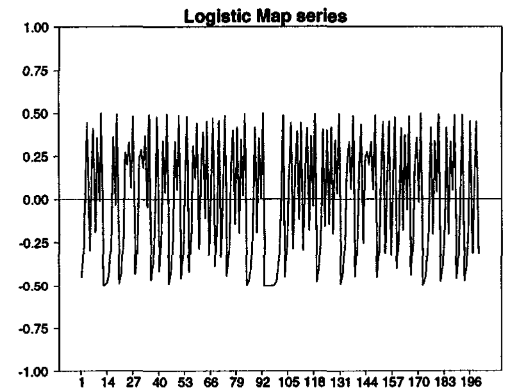
In order to get some idea of the implications of deterministic chaos we illustrate the case by comparing a truly deterministic chaos series with a random N(0,1) series. A logistic map model which takes the form $x_t = 4*x_{t-1}(1-x_{t-1})$ is used to generate the chaotic series. Both series contain 2000 observations; the initial value of the logistic map series is 0.3.[3] The figure on the following pages illustrates the time paths of these two series (only the first 200 observations are graphed), the respective autocorrelations for 60 lags, two dimensional plots in terms of the current and lagged value of the variable, the correlation dimension estimates with an embedding dimension 2–5 and the BDS test statistics with the embedding dimension 2 over the ε values 0.5–3.0.

It may be worth mentioning that all tests of chaos depend on the sampling procedure (time aggregation). Thus, if the high frequency chaotic data is measured only infrequently (for instance, daily observations are recorded only monthly), the data appear to be just random data. This property is illustrated by Figure 11 in the end of the paper (see also Table 4 for the BDS statistics in this case).

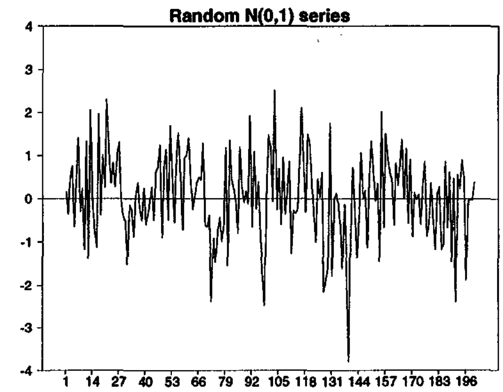
[3] The first value of the series is .300. The series are very sensitive with respect to this initial value. If the initial value is changed to .30001, the new series diverges from the original series after 14 observations and never converges. In addition to the logistic map specification we used the Henon map (which is equally often used a benchmark example). Here, the Henon map takes the following parametrization: $x_{t+1} = 1 + y_t + 1.4*x_t^2$ and $y_{t+1} = .3x_t$ with x(0) = .1 and y(0) = .1. Needless to say, these series also depend very much on the initial values. Both the logistic map and the Henon map are obviously very simple illustrations of chaotic behaviour and they should not be considered as representative models.

Figure 1. **Comparison of logistic map and random series**

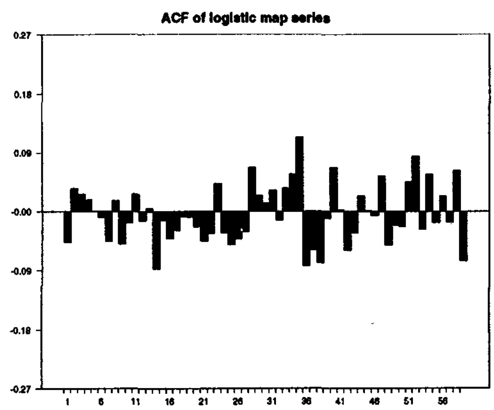
Logistic map series (adjusted with mean)

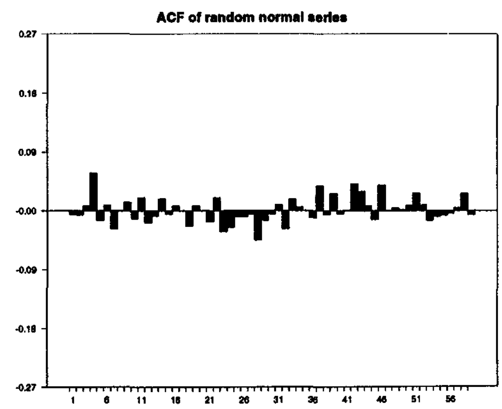


Random N(0,1) series



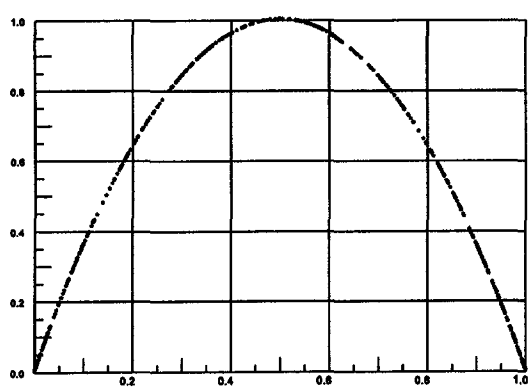
Autocorrelations



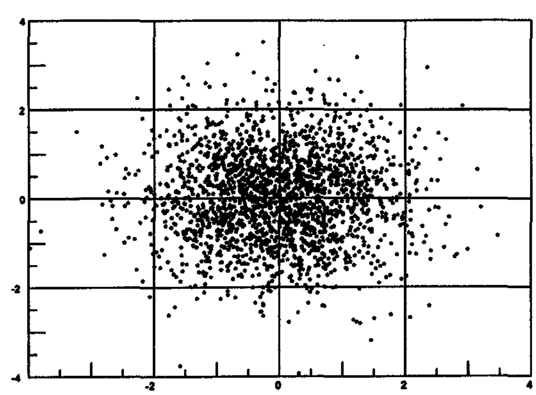


Two-dimensional plots

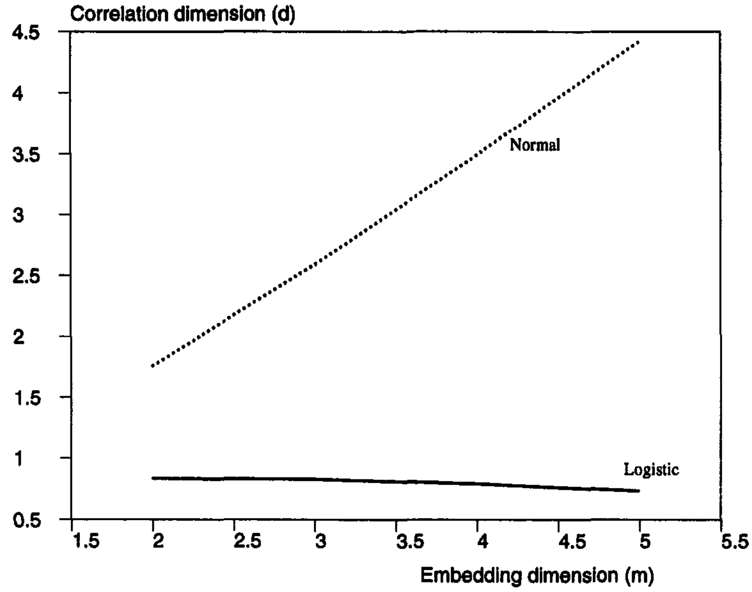
Logistic map series



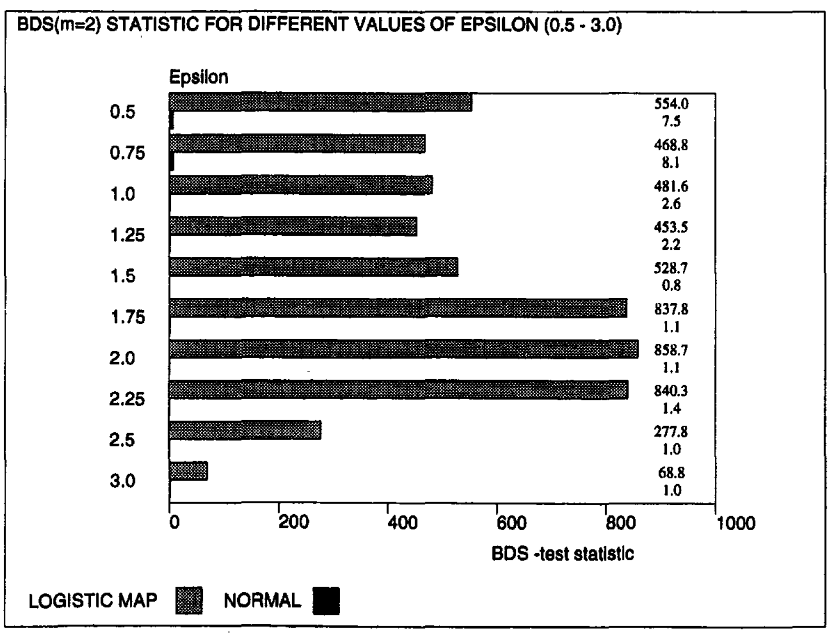
Random N(0,1) series



Correlation dimensions of logistic map and random normal processes

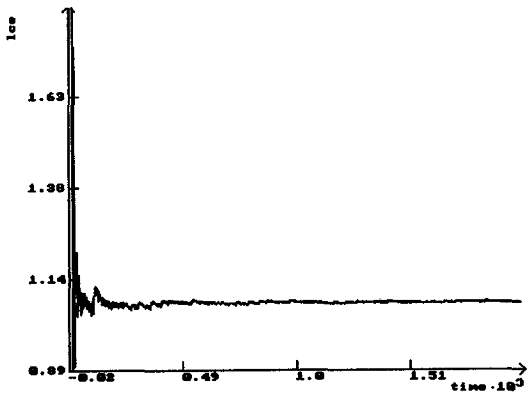


BDS(2) statistics for ε = 0.5-3.0

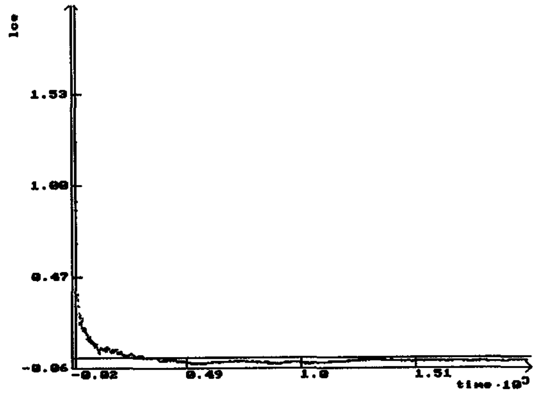


Lyapunov exponents

Logistic map, $L_1 = 1.07$

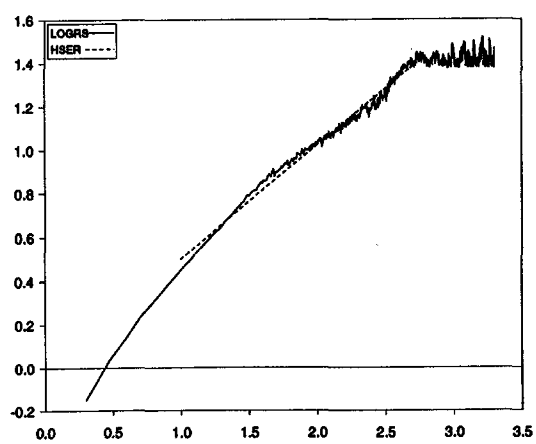


Random N(0,1), $L_1 = -0.02$

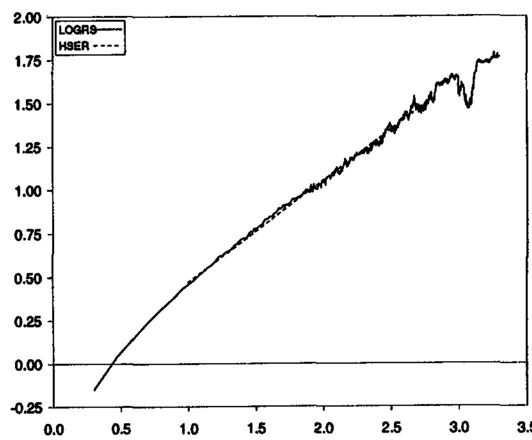


Hurst exponents

Logistic map, H = .426 (.520/.024)



Random N(0,1), H = .588



The purpose of Figure 1 is to show that the time series and the autocorrelations are quite similar. In fact, one might at first glance consider the logistic map series to be random walk series. The dimension plots show, however, that there is a fundamental difference between these two series. The random N(0,1) series is spread quite evenly over the plane while the logistic map series does not fill enough space at a sufficiently high embedding dimension, which is a generic property of chaotic processes. The clustering of two-dimensional plots also shows up in the dimension estimates (and in the BDS test statistics). The estimate for the logistic map series is about one irrespective of the embedding dimension (it can be shown that the correlation dimension for the logistic map is 1.00 ±0.02, see, e.g., Hsieh (1991)). Finally, the BDS test statistics clearly discriminate these two series. Thus, the statistic for random normal series typically fails to exceed the critical value while the test statistic for the logistic map exceeds the critical value by many hundreds.

Lyapunov exponents

The Lyapunov exponents measure the average stability properties of the system on the attractor. Frequently the presence of at least one positive Lyapunov exponent is taken to be the definition of chaos. For a fixed point attractor, the Lyapunov exponents are the absolute numbers of the eigenvalues of the Jacobian matrix evaluated at the fixed point. Thus, the Lyapunov exponents can be considered as generalizations of eigenvalues (see, e.g. Medio (1992) and Frank and Stengos (1988a) for further details).

To define the Lyapunov exponents consider the following Nth order dynamic system:

$$\frac{dx}{dt} = F(x),$$

where x is a vector with N components. Consider a trajectory x*(t) that satisfies this equation and an arbitrarily small positive initial displacement from the start of x*(t) denoted by D(t). Now, it can be shown under fairly general conditions that, for given D(0), the following limit exists:

$$L_i = \lim_{t\to\infty}(t^{-1}\ln|D_i(t)|), \ t = 1,2,...N.$$

Notice that the Lyapunov exponents are not local properties as one might think. Thus, the values of L_i are independent of the choice of D(0). In fact, one may interpret the exponent(s) to measure the average rate of separation over the entire strange attractor.

A positive Lyapunov exponent measures how rapidly nearby points diverge from each other. A negative Lyapunov exponent, in turn, measures how long it takes for a system to reestablish itself after it has been perturbed. Basically, this is the reason why the Lyapunov exponents offer a way to classify attractors.

The problem is that it is not easy to estimate Lyapunov exponents from experimental data. Wolf et al (1985) have developed a FORTRAN program which estimates the largest exponent L_1 from these kinds of data but it has been shown (see, e.g., Brock (1986) and Brock and Sayers (1988)) that the estimates are very sensitive with respect to the nuisance parameters used in the context of the program. Thus, for instance, large positive Lyapunov estimates may be obtained for pure noise data. Our own experience points in the same direction. Therefore we are reluctant to use the Wolf et al (1985) estimates to characterize our real data.

Quite recently, McCafferty et al (1991) and Dechert and Genacay (1993) have proposed an alternative algorithm using the so-called multilayer feedforward networks, which appear to have superior properties with respect to the Wolf et al (1985) algorithm. This will allow us to rescrutinize the values of the Lyapunov exponents in a more affirmative way. Although we do not go through the analysis of Lyapunov exponents with the real data, we may refer to Figure 1 in the text where the largest Lyapunov exponent is presented for random normal and logistic map time series.[4] In the case of logistic map series, the exponent is large and positive while with random noise series the exponent converges to a (small) negative value.

The ARFIMA model estimates

Long-term memory often shows up in the form of nonperiodic cycles. This has lead to development of stochastic models that exhibit dependence even over very long spans, such as the fractionally-integrated time series models. The models have autocorrelation functions which decay at much slower rates than those of weakly dependent (mixing) processes. This example, the data generating process of X_t could be the following

$$(1-L)^d X_t = e_t$$

where L is the lag operator and e the white noise term. d can be noninteger which gives a "fractionally differenced" (or order d) time series. Now, if X_t is stationary and invertible for $d \in (-½,½)$ and exhibits a unique kind of dependence that is positive or negative depending on whether d is positive or negative. If d is positive autocorrelation decay very slowly, indeed so slowly that their sum diverges to infinity. If $d<0$, they sum collapses to zero, instead (see e.g. Lo (1991) for demonstration of the effects of fractional differencing on the autocorrelation function).

Estimating an ARFIMA model is not a easy task. Computational problems are not of second order importance. In addition, one has to find out the proper

[4] Lyapunov exponents have been estimated in several empirical studies; see, e.g., Frank and Stengos (1988c), Frank et al (1988) and Peters (1993). The results have been somewhat mixed, partly depending on the algorithm (thus, for instance, Frank and Stengos (1988) do not find support for the existence of chaos while Peters's results point in the opposite direction). There is, however, a lot of ambiguity concerning the results because of convergence problems and computational sensitivity.

specification for the estimating model. Here this problem boils down in determining the lag structure for the AR part of the model.

The Hurst exponent (rescaled range analysis)

The Hurst exponent is a new measure which can classify time series in terms of persistence (or "antipersistence"), stability of the data-generating mechanism and the importance of outlier-type observations. Thus, it can distinguish between a random series and a non-random series, even if the random series is non-Gaussian. The Hurst exponent was first applied to natural systems (first, in analyzing water reservoir control within the Nile River Dam project in early 1900) but recently there have been numerous applications to financial data (see, e.g., DeGrauwe, et al (1993) and Peters (1993)).

Computing the Hurst exponent (H) and the related V test statistic requires the following steps:

$$Q_n \equiv \frac{1}{\hat{\delta}_n(q)}\left[\max_{1\le k\le n} \sum_{j=1}^{k}(X_j-\bar{X}_n) - \min_{1\le k\le n} \sum_{j=1}^{k}(X_j-\bar{X}_n)\right],$$

where

$$\hat{\delta}_n^2(q) \equiv \frac{1}{n}\sum_{j=1}^{n}(X_j-\bar{X}_n)^2 + \frac{2}{n}\sum_{j=1}^{q} w_j(q)\left\{\sum_{i=j+1}^{n}(X_i-\bar{X}_n)(X_{i-j}-\bar{X}_n)\right\}$$

where X is the sample mean $(1/n)\sum_j X_j$. $\hat{\delta}_n^2(q)$ is simply the square root of consistent estimator of the partial sum's variance. If there is no short-run dependence, the variance is simply the variance of the individual terms X_t. In the presence of short-run dependence we have to modify the statistic (following Lo (1991)) and include also the autocovariance terms.

Under the i.i.d null hypothesis, as n increases without bound, the rescaled range Q_n converges in distribution to a well-defined random variable V when property normalized so that

$$\frac{1}{\sqrt{n}} Q_n \Rightarrow V$$

The fractiles of the (non-modified) V statistic are reported in e.g. Lo (1991). As for the Hurst exponent, it can be estimated from the following model

$$Q_n = (\alpha \cdot n)^H$$

where α is the scaling constant.

In detecting long-run dependence, the rescaled range analysis is probably not the most efficient way of doing that. By contrast, estimating the fractional differencing parameter directly (e.g. in the context of an ARFIMA model) would be a better way. Still, the R/S analysis could be useful as a complementary tool in assessing more general features of long-run dependence (for further details, see., e.g., DeGrauwe et al (1993) and Peters (1993)).

According to the statistical mechanics, H should equal 0.5 if the series is a random walk. In other words, the range of cumulative deviations should increase with the square root of time. For many (most?) time series from a natural system, the value of H has turned out to be much higher than 0.5. In surprisingly many cases the value of 0.73 is obtained (see DeGrauwe et al (1993)).

When H is different from 0.5, the observations are no longer independent in the sense that they carry a memory of all preceding events. This memory can be characterized as "long-term memory". Theoretically, it lasts forever. Thus, the current data reflect everything which has happened in the past. Notice that this is something which cannot be taken into account in standard econometrics, where time invariance is assumed.[5]

Now, consider the case where $H < ½$ and $H > ½$. In the former case, the system is antipersistent or "mean reverting". Thus if the system has been up in the previous period, it is more likely to be down in the next period. By contrast, when $0.5 < H < 1$, the system is persistent or "trend-enforcing". If the series has been down in the last period, then the chances are that it will continue to be down in the next period.

A R/S plot for random N(0,1) and a logistic map series is presented in Figure 1 in the text. Note that the estimated slope (i.e., the Hurst exponent) is 0.59, which is quite close to the theoretical value of 0.5. For finite series, the expected value of H, E(H), is in fact somewhat larger than 0.5. Thus, the value of 0.59 may well fall inside the confidence interval of E(H) (Var(H) = 1/n; see Peters (1994)). The estimated slope of the logistic map series is instead 0.43 (as for the Henon map, an estimate of .38 is obtained; for the logistic map series see Figure 1, see also Peters (1994)), which says (in statistical terms) that this series has no population mean and that the distribution of variance is undefined. Clearly, there is nothing we can forecast with these series.

The Ramsey irreversibility test

The irreversibility test, which has been derived by Ramsey and Rothman (1988) and Rothman (1993), deals with the concept of time reversibility.[6] Time

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[5] The values of the Hurst exponent can be related to a correlation (C) measured in the following way: $C = 2^{(2H-1)} - 1$. Thus, when $H = ½$, $C = 0$, and we are dealing with a random series. Its probability density function may be the normal curve but it does not have to be. By contrast, if H is different from ½, the distribution is not normal.

[6] A stationary time series $\{x_t\}$ is time reversible if for any positive integer n, and for every $t_1, t_2, ..., t_n \in z$, where z is the set of integers, the vectors $(x_{t1}, x_{t2}, ..., x_{tn})$ and $(x_{-t1}, x_{-t2}, ..., x_{-tn})$ have the same joint probability distributions. A stationary time series which is not time reversible is said to be irreversible. Notice that, by definition, a non-stationary series is time irreversible. See e.g. Tong (1983) for further details.

irreversibility is a concept which is useful in analyzing possible asymmetries (nonlinearities) in economic time series, for instance, in output series. According to the conventional Mitchell-Keynes business cycle hypothesis, cyclical upturns are longer, but less steep, than downturns (see also the "plucking model" of Friedman (1993)) If one traces out the behaviour of cycles in reverse time it can be seen that the symmetric cycle is time reversible and that the asymmetric cycle is time irreversible.

Ramsey and Rothman (1988) propose that the presence of time irreversibility should be checked by estimating a symmetric bicovariance function in terms of x_t. The test statistic which is obtained from this bicovariance function is of the following type:

$$G_{i,j}^{k} = T^{-1}\sum_{t=1}^{T}\left[(x_{t1})^{i}(x_{t-k})^{j} - (x_t)^{j}(x_{t-k})^{i}\right], \qquad k = 1,2,\ldots,K.$$

If the time series is time reversible, $G_{i,j}^{k}=0$ for all k. As for the choice of exponents, i and j, we assume here that i = 2 and j = 1 (here we just follow Ramsey (1990)). In addition, we experiment with the pair i = 3 and j = 1. The maximum lag length K is set at 120. To ensure stationarity, we also use here AR(4) residuals instead of the original time series. The significance of the G statistic is tested by computing the confidence limits according to the following formula for the variance of $G_{1,2}^{k}$:

$$\mathrm{Var}\left[G_{1,2}^{k}\right] = \left(\frac{2}{(T-k)}\right)\left[\mu_4\mu_2 - \mu_2^3\right],$$

where $\mu_2 = E\left[x_t^2\right]$ and $\mu_4 = E\left[x_t^4\right]$. Assuming that the data are independent and identically distributed $N(0,\sigma^2)$, the right hand side of the above formula can be simplified to $\left(\frac{4}{(T-1)}\right)\left[\mu_2^3\right]$. This is clearly a crude approximation because the normality assumption does not hold, nor are the variables uncorrelated. However, it is not at all clear how the variance terms should be computed when x_t is not i.i.d. but follows e.g. some general ARMA(p,q) model (see Ramsey and Rothman (1988) for various experiments). The test statistics and the respective confidence limits are displayed in Figure 8.

A nonlinear adjustment equation

Instead of just computing test statistics for nonlinearity, it would be tempting to estimate a general nonlinear time series model and compare its properties with a linear model. Unfortunately, such a general nonlinear model does not exist nor is there any agreement on a reasonable approximation which could be used to capture the possible nonlinear elements of the data. Still, the situation is not completely hopeless. There are some interesting candidates for a nonlinear specification. The first which deserves to be mentioned is the threshold model specification introduced by Tong (see e.g. Tong (1983)). Another specification which is clearly worth mentioning is the nonlinear employment (output) equation introduced by Pfann (1992). This (estimating) equation takes the following form:

$$x_t = a_0 + a_1 t + a_2 x_{t-1} + a_3 x_{t-2} + a_4 (x_{t-1} x_{t-2}) + a_5 (x_{t-1}^3 x_{t-2}) + a_6 (x_{t-1} - x_{t-2})^3 + \mu_t,$$

where μ is the random term. According to Pfann (1992) and Pfann and Palm (1993), the parameter of the nonlinear terms can be unambiguously signed in the case of employment equations. Thus, a_4 should be positive (if hiring costs are higher than firing costs, or in general, if the cycle spends more time rising to a peak than time falling to a trough). Moreover, parameter a_5 is expected to be negative if the asymmetry (skewness) of magnitude (i.e. the magnitude of troughs exceeds the magnitude of peaks) is negative and parameter a_6 is also negative if the asymmetry (skewness) of duration is negative (i.e., it takes longer for a series to rise from a trough to a peak than to fall from a peak to a trough).

Although this model may make more sense with (productive) input and output series, we also apply it to all ten (here, in fact, thirteen) Finnish series partly to see whether the real and nominal series can be discriminated on the basis of this equation. The results are reported in Table 9. This table also includes a comparison of this model with a linear alternative.[7]

4 Test results with univariate models

4.1 Results from diagnostic tests

The message of the empirical analyses is quite clear and systematic: the data do not give much support to linear models. Thus, all the test statistics reported in Tables 2 and 3 indicate that at least a linear AR(4) model is in trouble.[8] According to Table 2, the residuals from the AR(4) model suffer from heteroskedasticity and non-normality. The ARCH(7) statistic is significant for all variables (perhaps excluding the interest rate). Thus, even with real series like industrial output an autoregressive conditional heteroskedasticity effect can be discerned. This is something new. Nobody is surely surprised to find an ARCH effect in stock prices but here a similar result applies to other variables as well.

[7] Here, we merely replicate the experiments by Pfann (1992). Thus, we take the same detrending procedure (see the second term on the right hand side) and the same lag structure. Obviously, extending the lag length beyond 2 would enormously complicate the model.

[8] In addition to the test statistics reported in Table 2, we also computed the Keenan (1985) and McLeod-Li (1983) test statistics. Both of these turned out to be highly significant. Thus the marginal significance levels were in all cases well below 5 per cent. The test statistics were also computed for the post-Second Word War period. Results were quite similar to those reported in Table 2. Thus the war itself cannot explain why the results lend support to nonlinearities.

Non-normality is clearly a severe problem. It is quite obvious that normality is violated because of outlier observations. Clearly, some observations can be classified as outliers and it might well be that these observations contribute to the rejection of linearity. This can be seen from Figures 2 and 3 which contain the time series and frequency distributions for the AR(4) residuals. In accordance with Table 1, the main problem seems to be excess kurtosis, not so much excess skewness. Although the normality assumption is rejected, the graphs suggest that the distributional problems are not, after all, so severe as the Jarque–Bera normality test statistic suggests.

Unfortunately, there is no obvious remedy for non-normality and outlier observations. One alternative is, of course, to use robust estimators and examine whether the results (e.g., the properties of residuals) change importantly as a result of the change in estimators. In fact, we did do this but it turned out that the results with the least absolute deviations estimator were qualitatively very similar to the OLS results. Another possibility is to reconsider the relevant sampling distributions of the nonlinearity tests statistics in the light of observed behaviour of OLS residuals. Here, we have not yet worked out this alternative.

After these considerations, some comments on the RESET and TSAY nonlinearity test statistics merit note. Both tests do suggest that the (linear) functional form is misspecified for most of the variables. The results are, however, very systematic. Thus, for instance, industrial production and bankruptcies, on the one hand, and narrow money and credit supply, on the other hand, behave in a different way in these tests. Moreover, the test results do not allow us to draw a clear line between real and nominal variables. As far as Hsieh's (1991) third-order moment coefficients are concerned, one can see that with some variables the coefficients are very high. Some of the highest coefficients are, in fact, quite similar to those of the logistic map series! High coefficient values are obtained for the real exchange rate, consumer and wholesale prices, money and – somewhat surprisingly – stock prices. By contrast, the values for industrial production, bankruptcies and the terms of trade are somewhat lower although all of them are not "clean". Thus, nonlinearities do exist and nonlinearities are not only a problem for real variables. Since the third-order moment coefficients are not intended to test models which are nonlinear in variance, one may conclude that the high coefficient values for the nominal series not (only) reflect some ARCH effects but also other sorts of nonlinearities (say GARCH-in-Mean effects or long-memory behaviour).

4.2 Results from analyses of the correlation dimension

Next, we turn to results from the analysis of the correlation dimension. These results are presented as follows: First, the two-dimensional plots of the AR(4) residuals are presented in Figure 5, then the correlation dimension estimates are presented in Figure 6 (Figure 6 consists of two plots showing the correlation integral and the derivative of $C(\epsilon)$ in terms of ϵ; the respective numerical values are reported in Table 3) and, finally, the BDS test statistics are reported in Table 4.

Unfortunately, the results from these exercises are somewhat different. First, the dimension plots are not consistent with the existence of low-dimensional chaotic behaviour (notice, however, that we just look at things very informally in two dimensions). Although there are some differences between variables, none of the variables behaves in a chaotic manner. Stock prices may best correspond to a random variable (observations are evenly distributed over the x_t, x_{t-1} plane) while some clustering takes place in consumption and wholesale prices.

As one might expect on the basis of the dimension plots, the estimates of the correlation dimension (the embedding dimension running from 2 to 5) lend very little support to a model of chaotic behaviour. The estimate of d(m) increases almost linearly with the embedding dimension m. Only wholesale prices give an opposite result. The estimate of d(m) remains in the neighbourhood of one even if the embedding dimension is increased to 5. Figure 2 may explain why this result emerges. The behaviour of prices in the 1920s and 1930s was completely different from the rest of the sample period (i.e. the price level was practically stationary during the pre-war period while after the outbreak of the Second World War the rate of inflation turned out to be stationary). If the 1920s and 1930s are dropped from the sample, the correlation dimension estimates behave well in accordance with the other variables.[9]

Somewhat contrary to these results, the BDS statistics turn out to be very high, suggesting that the data-generating mechanism is not linear. The null hypothesis that the series are random i.i.d variates is rejected in all cases with standard significance levels. The same result emerges when ARCH residuals are used instead of OLS residuals. A completely different result emerges, however, when the series are shuffled, i.e. the observations are arranged in a random order. Then the null hypothesis of independent observations is typically not rejected, which suggests that the distributional assumptions are not very critical in terms of the outcome of the BDS statistics. By contrast, the time-series structure is the important aspect which produces the very high values of the BDS statistics.

But how should we interpret this conflicting evidence? Should more stress be given to the correlation dimension estimates or the BDS test statistics. The answer is not easy. Perhaps the best way to summarize this evidence is to conclude that there are definitely some signs of nonlinearity but not necessarily of deterministic chaos.

4.3 Results from the tests for the long memory property

As pointed above, the analyses make use of the ARFIMA model estimates and the rescaled range test statistics. In addition, we carry out more informal tests by scrutinizing the autocorrelation structure of AR(4) residuals in terms of different transformations. This research menu may reflect the fact all of these analytical tools are used in assessing the presence of long-run dependence.

In time series, a long-term memory property is said to be present if absolute values of a stationary variable r_t have significant autocorrelations for long lags, i.e. $\rho(|r_{t-k}|, |r_t|) \neq 0$, when k is large. This property was first noted for speculative price series by Taylor (1986) and thereafter also called the Taylor effect (see

[9] For the period 1939M9-1996M9 the following set of dimension estimates was obtained: m = 2: 1.901 (1.02); m = 3: 2.709 (1.30); m = 4: 3.617 (1.94) and m = 5: 4.226 (1.01). These values are clearly in accordance with the other values in Table 3 and hardly consistent with the existence of deterministic chaos.

Granger and Ding (1993)). In practice, this property implies that the simple random walk model does not hold for stock prices, even if the price changes are serially uncorrelated. This phenomenon also shows up in the rescaled range analysis with the Hurst exponents. High ($H > 0.5$) values of the Hurst exponent imply strong persistence – the fact that the observations (residuals) are independent but they have a memory. Thus, the data are not generated with a random walk but, instead, with a biased random walk or, in other words, with fractional brownian motion.

For instance, if we consider stock price changes, it seems intuitively appealing to observe that they are uncorrelated, but this does not explain anything about the heteroskedasticity found in them. Statistically, stock prices could be martingales with non-constant innovation variance (see e.g. Spanos (1986)). However, from the economic point of view, the problem is to find out whether residual variance from the linear model follows conditional heteroskedasticity (ARCH), a generalized version of it (GARCH), asymmetric power ARCH (A-PARCH as defined in Ding, Granger and Engle (1993)) or some other form of heteroskedasticity appropriate for the particular time series. However, univariate models could be helpful in identification and prediction of the type of heteroskedasticity, but probably insufficient for understanding these processes.[10]

Heteroskedasticity in residuals already shows that stronger forms of rational expectations rationality, which imply efficient use of all information, does not hold for higher moments of the process. In fact, expectation errors are not white noise, but rather innovation processes with non-constant variance. The long-memory phenomenon also puts emphasis on the long-term cyclical swings often encountered in economic time series. These cyclical swings could relate to business cycles or even Kutznets and Kontrajev cycles or a tendency to generate serious financial crises like those withnessed in the 1930s and 1980s. However, as Granger and Ding (1993) emphasize, caution should be observed in interpretation, since it is not the series themselves but their absolute values that have the long-memory property.

If the efficient market hypothesis were to hold strictly, the random walk property would imply that r_t is an i.i.d process. In addition, any transformation of r_t, like $|r_t|$ or r_t^2 should also be an i.i.d process (Ding, Granger, Engle (1993), p. 87). The sample autocorrelations of the i.i.d process will have finite variance $1/\sqrt{(T)}$ and larger correlations for $|r_t|$ will indicate the long-memory property. Ding, Granger and Engle (1993) show that, if $|r_t|^d$ is taken as a yardstick in measuring the strength of autocorrelation for long lags, the long-memory property is strongest around $d = 1$.

In the same way as Ding, Granger and Engle (1993), we found that all variables in our data set showed clear evidence of long memory, and thus the sample autocorrelations for absolute values of residuals were greater than the autocorrelations of squared residuals. This resemblance could indicate that economic time series have characteristics of models not fully described or understood so far.

[10] Granger and Teräsvirta (1993, pp. 51–53) note that a series may have short memory in mean, and long memory in variance, but that the opposite is not so likely, i.e. long memory in mean with short memory in variance. Short memory in mean is often found in stationary series, whereas long memory is present in integrated "level" series.

Series which had $|r_t|$ well above r_t^2 were industrial production, the terms of trade, the real exchange rate and the interest rate. Slightly different were series such as bankruptcies, wholesale prices, money supply (M2) and stock prices, which mostly shared the same characteristics. This could be due to rare, but large discrete changes in these series, such as e.g. the effects of devaluations and strikes. The results from these long-memory tests performed for AR(4)-residuals of our time series are presented in Table 5 below. Graphs of sample autocorrelation functions for the absolute values of the AR(4) residuals are shown in Figure 7.

Among other things, the results indicate that linear filtering with an AR(4) model is not sufficient to remove dependence on the distant past in these series, even though model selection criteria would suggest at most times that the fourth-order autoregressive polynomial should be long enough. Despite the fact that these series have dominant long-run features like unit roots and trends, parsimonious linear models seem unable to account for these. Observations therefore point to the conclusion that trends in economic time series are more likely to be stochastic than deterministic. Hence we come up against nonlinearities again.

The main message is, however, there is significant long-run dependence in all of the real and monetary series. In addition, there seems to be no clear difference between real and monetary variables as regards how fast autocorrelations would die out for long lags. Whether that dependence can be accounted for specific long memory models is analyzed next.

Now, turn to estimation results with ARFIMA models. Estimation is carried out using both original time series and fitting an ARFIMA(4,d,0) model to these series and using the AR(4) residuals and fitting an ARFIMA(1,d,0) model to these series. The ARFIMA/OX program by Doornik is used estimation. Both the Maximum Likelihood (ML) and the Nonlinear Least Squares (NLS) estimators are used here.[11]

The estimates are presented in Tables 6–8 below. Table 6 contains the NLS estimates for the ARFIMA(4,d,0) model for unfiltered time series (a log transformation is taken for all series except for tt, fx and r). The ARFIMA(1,d,0) estimates are reported in Table 7 (for the AR(4) residuals)) for both ML and NLS estimators. Finally, we report in the difference parameter estimates which have been obtained by using the Geweke-Porter-Hudak (1983) approach. The estimated fractional difference parameters correspond here again to the AR(4) filtered series.

All in all, the results lend some support to the long-memory property but the overall evidence is quite moot. In the case of unfiltered data, there are some examples (i.e., tt and r) in which the fractional difference parameter is significant. With filtered data, the evidence is much weaker (for quite obvious reaons). Turning to the Geweke-Porter Hudak differencing parameter estimates in Table 8, one may notice that they give a bit more evidence of fractional differencing. Results with money and credit series (in addition to consumption prices) suggest that the differencing parameter may indeed be fractional and thus the series may be long-run dependent. What makes this observation important is the fact the subsequent results with rescaled range analysis point to the same direction.

[11]There were several computational problems with the Maximum Likelihood estimation, i.e. computational failures and great sensitivity with respect to initial values. The nonlinear least squares alternative performed much better in this respect.

The results for the rescaled range analysis are reported in Table 9 and in Figure 8. Table 8 contains the V statistics for all time series in addition to the estimates of the slope parameters, i.e. the Hurst exponents H. The time series graphs for Q are presented in Figure 8 to illustrate stability of pattern of (possible) short and long dependence.

Looking at Table 8 shows that the Hurst exponent is generally above 0.5. One has, however, to take into account the fact that in finite samples the expected value of H is well above 0.5 (see Peters (1994) for simulated values of E(H)). Thus, if one scrutinizes the values of the test statistic V, it comes out that they are generally not statistically significant. In other words, there is only weak evidence of long memory. The series which show long-memory properties are in fact quite the same which showed similar properties in the case of Geweke-Porter-Hudak estimator. Thus, money and credit series together with prices behave in a manner which is consistent with long memory. For all other series, the evidence is less compelling - and for "real" series there is no evidence of long memory.[12]

An interesting question is how long is the long-memory phenomenon. Is there a certain time span - say one year - during which observations are not independent. One could, for instance, argue that for various reasons (see, e.g., Peters (1993)) the stock market is not efficient in the short run but efficient in the long run. In other words, we have some cycles which just reflect these inefficiencies (or, more generally market imperfections). This might show up in a change of the R/S slope. In fact, this kind of reasoning seems to apply to the Finnish stock price series. There is quite a clear change in the slope after 300 data points corresponding to a cycle of about 12 years. In the short run, stock prices seem to follow the random walk model (H equals to .48) while in the long run stock prices can be characterized as independent or even antipersistent. Thus, an estimate of .18 is obtained for the data points exceeding 300 (see Figure 7). Clearly, this finding is consistent with the results obtained by Peters (1993) in terms of behavioral changes but equally clearly the finding is at variance with the long-memory property of stock prices.

4.4 Results from the time irreversibility analysis

A similar result emerges with Ramsey's (1990) irreversibility test statistics reported in Figure 8.1. Although, the confidence limits are only indicative, some signs of nonlinearities can be discerned with all series. Somewhat surprisingly, stock prices do not seem to be the most striking example of this sort of nonlinearity. Thus, for instance, the test results for industrial production tell more about nonlinearities than the results for the stock index (see Figure 8.2). Bankruptcies and banks' total credit supply seem to be more obvious candidates. Perhaps this is something which is in accordance with the observed nature of

[12]The results changed only marginally when the original rescaled range statistic was replaced by the modified rescaled range statistic. As with Lo (1991), the values of V did generally decrease although the changes was on average quite small. More important change took place when the AR(4) residuals were replaced by first (log) differences. The evidence on long memory was in this case even weaker than with the AR(4) residuals.

indebtedness and the relationship between indebtedness, credit supply and bankruptcies (see, for instance, Stiglitz and Weiss (1981) and Bernanke (1983)).

Recently, Rothman (1993) has shown that the Ramsey-Rothman irreversibility test is relatively powerful against the threshold model. Thus, our findings could also be interpreted from this point of view. In other words, there are nonlinearities but not of deterministic chaos type but rather resulting from nonlinear model structure or parameter instability. In the subsequent sections, we consider these alternatives.

4.5 Estimates of adjustment equations

Can anything else be said about the nature of nonlinearities? Tables 2 and 6 suggest that this is the case.[13] Table 1 indicates that the real series and the nominal series behave in a very different way. The nominal series do not show any signs of negative skewness. Moreover, the nonlinear adjustment equations (reported in Table 6) behave very badly, for instance, in terms of stationarity.[14] It is particularly interesting to compare the behaviour of industrial production and stock prices. Industrial output is characterized by clear negative skewness (in magnitude) while there is no apparent skewness in stock prices. With industrial production, positive residuals are much smaller and obviously more numerous than negative residuals. Intuitively, this makes sense since capacity constraints limit increasing production while a decrease in orders or bankruptcies may lower production more rapidly. With stock prices, there is no difference between positive and negative residuals. Thus, adjustment of stock prices does not contain significant asymmetries. See Figure 9 for details; notice that positive and (absolute values of) negative AR(4) residuals are presented here in an ascending order.

4.6 Results from stability analysis

The adjustment properties could, of course, be scrutinized in a straightforward way by looking at the parameter stability over depressions and booms. Table 10 contains some indicators of parameter stability for the univariate AR(4) which is

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[13] Here, we have introduced three additional real variables: the real interest rate and the (inverses of) money and credit velocities.

[14] With consumer and wholesale prices, there seems to be positive skewness indicating that prices tend to increase faster than they tend to decrease, which obviously makes sense. The behaviour of the long-term interest rate may only reflect this same fact. The real exchange rate, in turn, is characterized by gradual deterioration of competitiveness and once-for-all devaluations of the currency. Money and credit seem to behave in the same way as stock prices in terms of skewness although the estimations results are somewhat different. With bankruptcies, the results represent some sort of puzzle. Industrial output and bankruptcies do not seem to be just mirror images – quite the contrary. Thus, there are some (although not very significant) signs of negative skewness indicating that peaks in bankruptcies are smaller than the corresponding troughs. This clearly indicates that bankruptcies are perhaps more related to financial and institutional variables than just to demand and output.

used as some sort of point of departure in this study. Thus, we have computed the average lag length for depression (the shaded areas in Figure 1) and non-depression periods, the Chow stability test statistic in terms of the sample split and an F-test statistic for the significance of multiplicative (x_{t-i}*depression dummy) terms. It turns out that the stability property is at variance with the data. Moreover, there is some, although not very strong, evidence of asymmetric adjustment in the sense that the average lag length is shorter in depressions than in "normal years".

The stability measures are to some extent consistent with the evidence from the nonlinear adjustment model but some clear inconsistencies also arise. For instance, somewhat conflicting results are obtained for bankruptcies and stock prices. It should be noticed, however, that the classification of observations is based on output behaviour and the cyclical behaviour of other variables, such as stock prices, do not coincide with output movements and, therefore, the results cannot be identical.

Thus, if anything can be learned from this exercise, it is the fact that nonlinearities do seem to exist with the long Finnish series but there are clear differences between nominal and real variables. Thus, it is perhaps futile to analyze all sorts of nonlinearities using a single model as a frame of reference.

5 Testing dependencies between residual moments

The purpose of first applying an autoregressive model to the series is to remove the potential trend component from them. The deterministic or stochastic long-term trend could be removed in other ways as well, e.g. by differencing or modelling with structural time series models and then eliminating the trend component. We proceed by calculating dependency measures of different transformations of these AR(4) residuals.[15] Different moments of residual series and absolute values of residuals are considered as transformations. Therefore we calculate dependence tests from cross-autocorrelations between these univariate residuals as a first step in searching for dynamic relationships.

As can be seen, this procedure looks like an extension of the Granger causality test. However, we start by calculating Portmanteau test statistics without conditioning on past observations of the transformed residuals of the series itself. Portmanteau tests give us potential evidence about the direction and strength of the dynamic dependencies between variables. If the relationship is one-sided, it greatly simplifies the identification of the sources of shocks in these series.

To test whether residuals of the autoregressive model satisfy the properties of independent white noise, this can be seen by calculating the Portmanteau (Q) statistic. This test is designed to detect departures from randomness among the k first auto- or crosscorrelations. The test has the following form

$$Q = T(T+2) \sum_{k=1}^{M} (T-k)^{-1} r_k^2,$$

[15] We also computed the same measures with respect to the ARCH-model residuals. The results turned out to be so close to the results with squared OLS residuals that we do not report them.

where r_k^2 are the squared correlations of the residuals.

This modification of the basic Box-Pierce statistic was first presented in Ljung and Box (1978). The test statistic is asymptotically $\chi^2(M)$ distributed when the original residuals are independent. There is no clear solution in choosing M, but in our case too small values could result in failure to detect dependencies between important higher order lags. As might be expected, increasing M will, on the other hand, lead to lower power of the test (Harvey (1981), p. 211).

The Portmanteau statistic could also be applied to the higher moments or absolute values of stationary series as a general test against non-randomness. McLeod and Li (1983) have shown that squared residuals have the same standard asymptotic variance (1/T) as the original series if the residuals are random. In the following tests we assumed the lag order of 24 (2 years) to be large enough to pick up long term dependencies between different moments of residuals. In our application economic theory has rather little to say about the lags between shocks leading to variation in other variables.

Table 11 presents a summary of the estimated Q test statistics. Only the number of significant cases is reported here. The test statistics have been computed both for leads and lags to get some idea of causality. A more detailed report of the results from cross-correlation analysis is available upon request from the authors.

With reference to the table we point out that in general the number of significant values is very high. Almost two-thirds of the coefficients are significant at the 5 per cent level of significance. Particularly in the case of absolute values of the AR(4) residuals, the dependencies are very strong. In accordance with the results from univariate long-memory tests, the results in Table 11 suggest that the long-memory phenomenon also applies to co-movements of different variables – and not only within real and nominal variables but between all macroeconomic variables.

As for the role of different variables, one may note that the bankruptcy variable is very important in terms of the correlation structure. In fact, the number of significant correlations for bankruptcies is bigger than with all other variables. By contrast, the money supply series M1 and M2 and the terms of trade tt are only moderately correlated with other variables.

The test results do not tell very much about causation. In general, the cross-correlation coefficients are of the same magnitude with respect to leads and lags. Therefore, it is very hard to draw any far-reaching conclusions on this matter.

Calculating the contemporaneous correlations between variables does not have any dynamic causal interpretation as it only indicates instantaneous linear co-movement (positive or negative) within a month. As can be seen from Table 11, about one-third of the off-diagonal correlations are significant at the 5 per cent level. The interpretation of (significant) correlations is in most cases rather straightforward. Thus, for instance, consumer prices correlate in an expected way with, wholesale prices, monetary variables like credit, money aggregate, stock prices and the real exchange rate but not with other real variables.[16]

Altogether, the correlations between higher moments of the AR(4) residuals - in the same way as between the absolute values - are so strikingly high that further analysis in a multivariate nonlinear set-up is clearly required. The first step is simply to find out why volatility changes are so much related. In addition, one has to think about a possible explanation to the observed strong co-skewness between variables. Finally, one has also to take into account the fact that the long-memory property also seems to apply to the co-movements of different series - both nominal and real. It seems at least that a (multivariate) ARCH model is not a sufficient or a proper specification to account for these features of the data.

6 Concluding remarks

The empirical analyses presented in this paper have given strong and unambiguous support to the existence of nonlinearities in Finnish historical time series. The univariate case is very clear but it seems that nonlinearities may be even stronger and more important in the multivariate set-up. Obviously, this calls for further research in this area.

It is surely not surprising that the exact nature of nonlinearities cannot be identified. We are inclined to conclude that deterministic chaos is not the probable explanation. It is to be noted that Brock and Potter (1993) arrive at a similar conclusion when they review some recent evidence from macroeconomic and financial data. Another explanation which is often mentioned in this context concerns ARCH and GARCH effects. It is typically found that, after these effects are accounted for, the evidence for nonlinearity and chaos is weakened (see, e.g., Hsieh (1991)). In this study, we found the ARCH effect to be of minor importance. Thus, the explanations for nonlinearities must be sought elsewhere. Nonlinearities may, for instance, reflect neglected nonstationarities but we would prefer to argue in favour of the specific (asymmetric) properties of the short-run (cyclical) adjustment process. There could well be various institutional arrangements and constraints, informational deficiencies, capacity constraints and so on which prevent immediate and symmetric adjustment and which, in turn, explain the empirical findings. Finally, various stability tests clearly indicate that the behaviour of macroeconomic variables is quite different in recession and expansion periods.

It seems highly possible that nonlinearities may change some widely accepted assumptions or results. Thus, for instance, the neutrality of money may not be so good a approximation as it seems in the context of linear models. It may also be that the conventional symmetric adjustment mechanisms represent a very poor framework for dynamic specification. Also, the short and long-run properties of

[16] On the other hand, it is interesting to note that wholesale prices do correlate with both real and monetary variables. Industrial production correlates only with wholesale prices and bankruptcies, but in both cases the sign of the correlation seems to be the opposite than expected. It is also hard to interpret why interest rates correlate positively with stock prices. According to present value formulae, the relation should be just opposite.

different time series and the way in which the corresponding markets function need to be carefully rethought in the light of, for instance, the long-memory results obtained in this study. Finally, it may be that the importance of certain variables (and unimportance of the other variables) in the propagation mechanism of nominal and real shocks in the economy will change a lot if nonlinearities are taken into account. The Finnish data suggest that, for instance, bankruptcies are such a neglected variable.

Table 1. **Descriptive statistics for the residuals of a linear AR(4) model**

	skewness	kurtosis	median	med(-)	med(+)	stand.dev.
ip	-.66	5.52	.222	-2.058	2.443	.056
bank	-.59	4.70	.747	-12.968	14.141	.306
tt	.66	26.37	.037	-21.609	23.849	2.263
fx	4.18	69.66	-.142	-.725	.540	3.010
r	.45	16.87	.002	-.026	.032	.274
cpi	3.34	30.36	-.126	-.460	.324	.013
wpi	3.04	21.42	-.115	-.506	.325	.012
credit	.04	8.38	.016	-.428	.450	.011
M1	.22	12.55	-.183	-1.612	1.372	.029
M2	-3.08	51.47	-.035	-.525	.472	.011
sx	-.13	5.09	-.096	-2.699	2.337	.050
st	-.085	2.73	-.455	-20.621	21.848	.366

Skewness and kurtosis denote the coefficients of skewness and kurtosis, respectively. Median denotes the sample median, med(-) and med(+) denote the 25 and 75 per cent (quartile) values. In the case of log transformation, the values of the median, med(-) and med(+) have been multiplied by 100. ip denotes (log) industrial production, bank (log) bankruptcies, tt the terms of trade, fx the real exchange rate index, r yield on long-term government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 (M2) the (log) narrow (broad) money, sx the (log) Unitas stock price index and st the turnover in stock exchange. The sample period is 1922M5-1996M9. (1) Not significant at the 5 per cent level.

Table 2. **Diagnostic test statistics for a linear AR(4) model**

	ARCH	RESET2	RESET3	Func. form	WHITE	J-B	TSAY
ip	20.65	0.65	1.72	14.61	11.72	306.88	10.94
bank	18.34	4.51	7.50	15.55	25.02	255.34	27.22
tt	10.69	7.60	8.76	3.74	4.64	2632.10	31.61
fx	8.24	4.18	8.84	2.89	4.70	1121.00	86.74
r	2.91	0.73	0.84	2.26	1.09	1636.60	23.55
cpi	41.43	3.39	3.72	23.05	13.64	473.89	106.39
wpi	8.06	4.36	5.22	8.60	9.44	563.05	29.67
credit	22.68	16.25	8.12	7.14	14.08	746.08	46.96
M1	20.57	5.17	5.96	24.64	9.13	1204.4	137.35
M2	2.94	17.79	10.73	17.34	9.22	1661.2	122.35
sx	44.23	0.12	0.37	41.73	6.88	380.38	48.10
st	17.02	0.46	6.45	18.15	9.26	154.74	52.55

ARCH denotes Engle's ARCH test statistic (with 7 lags), RESET2 test statistic adds the second power of the fitted value as an additional regressor RESET3 includes both the second and third powers of y. Func. form is the F-test of the second power of the explanatory variables and their cross-terms included in the regression. White denotes White' heteroskedasticity/functional form test statistic, J-B the Jarque-Bera test statistic for residual normality and TSAY Tsay's nonlinearity test statistic for 4 lags. 1 % and 5 % denote the critical values of the respective test statistics.

Table 2. **continued**

	r(1,1)	r(1,2)	r(1,3)	r(1,4)	r(2,2)	r(2,3)	r(2,4)	r(3,3)	r(3,4)	r(4,4)
ip	-.142	.112	-.011	-.119	.114	-.006	.031	-.144	.019	.086
bank	.194	.005	-.101	.021	.115	-.115	-.049	.206	-.019	.192
tt	-.237	-.002	-.123	-.041	.106	.032	.018	-.262	-.083	-.027
fx	-.494	-.370	-.404	.152	-.560	.345	-.634	-.547	.193	-.351
r	-.237	-.049	.013	.038	-.039	-.056	-.046	-.142	-.121	-.418
cpi	.619	.393	-.498	-.598	-.042	-.019	.353	.007	.796	.796
wpi	-.353	.113	-.118	.001	.137	.052	.302	-.378	.124	.044
credit	.124	-.147	-.212	.055	.112	-.113	.148	.069	.198	.009
M1	-.495	-.089	.313	.134	-.837	-.040	.266	-.638	.035	-.297
sx	.298	.188	-.038	.015	-.429	-.133	.148	.058	.115	-.031
random N(0,1)	-.040	-.015	-.011	-.016	-.005	.020	-.050	-.055	-.039	-.015
logistic map	.669	.536	.556	.558	.848	.544	.561	.833	.669	.536

r_{ij}'s are Hsieh's (1991) third-order moment coefficients $[\Sigma x_t x_{t-i} x_{t-j}/T]/[\Sigma x_t^2/T]^{1.5}$.

Table 3. **Estimates of correlation dimension with AR(4) residuals**

	Embedding dimension				
	1	2	3	4	5
ip	1.22	1.91	2.73	3.53	4.48
	(0.10)	(0.11)	(0.16)	(0.17)	(0.31)
bank	1.37	2.07	2.84	3.78	4.82
	(0.55)	(0.53)	(0.46)	(0.68)	(1.05)
tt	1.26	1.99	2.74	3.35	3.94
	(0.88)	(1.03)	(1.14)	(1.00)	(0.97)
fx	1.06	1.71	2.45	3.32	4.22
	(0.30)	(0.50)	(1.00)	(2.37)	(4.99)
r	0.55	0.57	0.59	0.62	0.63
	(5.51)	(4.12)	(2.62)	(2.37)	(2.29)
cpi	1.01	1.77	2.52	2.23	3.80
	(0.16)	(0.28)	(0.40)	(0.40)	(0.68)
wpi	0.81	1.28	1.37	1.40	1.42
	(10.08)	(28.56)	(82.97)	(144.18)	(188.76)
credit	1.00	1.75	2.59	3.50	4.39
	(0.18)	(0.33)	(0.54)	(1.03)	(1.55)
M1	1.14	1.87	2.70	3.65	4.69
	(0.14)	(0.15)	(0.15)	(0.27)	(0.56)
M2	1.22	1.97	2.83	3.75	4.81
	(0.10)	(0.10)	(0.11)	(0.19)	(0.48)
sx	1.17	1.92	2.79	3.61	4.25
	(0.17)	(0.18)	(0.22)	(0.12)	(0.24)
st	1.12	1.86	2.72	3.62	4.56
	(0.21)	(0.23)	(0.25)	(0.30)	(0.41)
random N(0,1)	1.05	1.85	2.80	3.78	4.61
	(0.14)	(0.11)	(0.16)	(0.26)	(0.09)
Henon map	0.97	1.27	1.29	1.30	1.32
	(0.01)	(0.03)	(0.02)	(0.05)	(0.05)
Logistic map	0.83	0.90	0.96	0.98	1.00
	(0.14)	(0.04)	(0.01)	(0.02)	(0.08)

Numbers inside parentheses are chi-square test statistics for the goodness of fit.

Table 4. **BDS test statistics for the residuals of a linear AR(4) model**

	m=2 ϵ=0.5	m=3 ϵ=0.5	m=4 ϵ=0.5	m=5 ϵ=0.5	m=2 ϵ=1.0	m=5 ϵ=1.0
Original AR(4) residuals						
ip	14.08	20.75	27.47	37.04	12.26	22.62
bank	8.76	13.08	11.68	-6.37	9.95	6.66
tt	11.90	14.84	18.77	24.71	9.52	14.75
fx	13.42	16.29	19.18	22.53	11.48	12.67
r	14.42	17.84	20.74	23.45	10.99	13.83
cpi	11.09	15.31	18.49	22.44	10.45	15.09
wpi	7.33	10.07	12.49	16.02	8.85	12.28
credit	10.95	14.24	17.65	22.62	11.21	17.66
M1	3.28	4.67	5.71	6.45	4.91	7.78
M2	6.70	10.41	13.59	16.83	7.35	14.10
sx	8.01	9.40	10.92	12.25	8.37	13.05
st	2.00	2.66	3.69	4.54	1.82	4.75
random N(0,1)	0.6	0.2	-0.3	-0.4	0.7	0.1
Henon map	165.9	280.8	428.9	717.7	76.0	91.3
logistic map	669.1	881.1	1152.1	1570.0	282.0	250.3
logistic map(4)	2.0	2.0	1.8	-8.1	-1.3	2.0
logistic map(30)	0.7	-2.1	-1.8	-2.2	-1.6	-0.9
ARCH(4) residuals of an AR(4) model						
ip	10.43	14.35	16.16	18.06	5.06	10.40
bank	11.91	14.53	16.63	18.28	9.98	12.39
tt	3.19	4.30	5.57	5.77	1.96	5.34
fx	6.03	8.94	9.08	8.99	1.60	4.40
r	6.21	7.76	8.20	8.40	3.37	5.36
cpi	14.03	14.84	14.63	14.61	9.05	9.50
wpi	10.33	10.63	10.36	9.77	11.08	9.97
credit	13.46	15.25	15.78	16.26	11.31	12.73
M1	6.61	9.21	10.47	10.78	2.80	7.35
M2	6.10	8.44	9.04	9.12	7.55	8.38
sx	9.44	12.98	15.28	17.22	8.03	12.57
st	4.18	5.50	6.56	7.70	4.51	5.87
Shuffled AR(4) residuals						
ip	-2.2	-1.4	-1.0	0.4	-2.5	-1.3
bank	-0.8	-0.2	-0.3	0.4	-0.9	1.1
tt	1.6	2.1	2.0	1.9	1.9	1.7
fx	0.4	1.0	0.7	0.5	1.4	1.5
r	1.9	1.6	1.3	1.2	1.7	1.0
cpi	2.7	2.6	2.3	0.2	0.7	1.2
wpi	-1.0	-1.6	-1.5	-1.2	-1.3	-1.6
credit	0.4	-0.1	-0.6	-0.4	-0.6	-0.8
M1	-0.6	-0.4	-0.3	-0.7	-1.3	-0.2
M2	-1.2	-0.7	-0.7	-0.8	-0.9	-0.3
sx	1.0	1.1	0.6	0.7	1.8	1.6
st	-0.4	0.1	0.1	0.2	-0.9	-0.1
Henon map	0.6	0.3	0.2	0.3	0.1	-0.0
logistic map	1.0	1.3	2.2	0.2	-0.2	1.2

The test statistic is BDS = $T^{1/2}[C_m(\epsilon)-C_1(\epsilon)^m]/\sigma_m(\epsilon)$, where T = N - m + 1 and N = the number of observations, $C_m(\varepsilon)$ = the correlation integral = T^{-2}*[number of ordered pairs (i,j) such that $\| x_{t,m} - x_{s,m} \| < \epsilon$] where $x_{t,m}$ is the m-history of the time series x and $\sigma_m(\epsilon)$ is the respective standard deviation. Under the null that the series is independently and identically distributed, the BDS has a limiting standard normal distribution. Here, ϵ = 0.5 corresponds to ϵ = 0.5*{the standard deviation of the residual series}. ϵ = 1.0 is defined in the same way. The shuffled series are obtained by sampling randomly with replacement from the data until a shuffled series of the same length as the original is obtained. The sample period is 1922M5-1994M9. The generated series include 1000 observations. logistic map(4) indicates that every 4:th observation is picked up from a generated series which originally include 30000 observations.

Table 5. **Autocorrelation tests for different residual transformations**

Variable	Significance level of the Ljung-Box Q(60) statistic for residual transformation u_t	$\lvert u_t \rvert$	u_t^2	First-order autocorrelation coefficients for residual transformations u_t	$\lvert u_t \rvert$	u_t^2
ip	.000	.000	.000	-.007	.326**	.167**
bank	.000	.000	.000	-.026	.346**	.247**
tt	.000	.000	.000	.018	.189**	.038
fx	.430	.000	.159	.023	.362**	.088*
r	.000	.000	.000	-.010	.284**	.072*
cpi	.000	.000	.000	-.008	.397**	.308**
wpi	.001	.000	.000	-.006	.332**	.182**
credit	.000	.000	.000	-.013	.343**	.314**
M1	.000	.000	.000	-.007	.219**	.166**
M2	.000	.000	.000	-.008	.265**	.129**
sx	.000	.000	.000	.002	.253**	.161**
st	.000	.000	.000	-.018	.116**	.197**

* (**) = significant at the 5 (1) per cent level.

Table 6. **NLS estimates for the ARFIMA(4,d,0) model**

Variable	ip	bank	tt	fx	r	cpi
$\hat{d}$	-.0377	.8829	.4895	-.1098	.4523	.0873
	(.0327)	(.0483)	(.0872)	(.0799)	(.1930)	(.0316)
$\hat{\alpha}_1$	.6691	-.5650	.6249	1.1537	.4517	1.2517
	(.0462)	(.0566)	(.0926)	(.0866)	(.1933)	(.0450)
$\hat{\alpha}_2$	.1948	-.3134	.1327	-.0660	.2181	-.0383
	(.0404)	(.0597)	(.0382)	(.0682)	(.0366)	(.0578)
$\hat{\alpha}_3$	.1072	-.1826	.2744	-.1851	.1181	-.2125
	(.0378)	(.0516)	(.0378)	(.0467)	(.0478)	(.0327)
$\hat{\alpha}_4$	.0285	-.1255	-.2225	.0895	.0971	-.0013
	(.0274)	(.0387)	(.0374)	(.0284)	(.0606)	(.0064)
$\hat{\delta}^2$	.0031	.0906	4.8806	8.8599	.0957	.0002

Variable	wpi	credit	M1	M2	sx	st
$\hat{d}$	.1421	.0741	.0187	.1038	.0609	.9139
	(.0374)	(.0159)	(.0667)	(.0319)	(.0501)	(.0556)
$\hat{\alpha}_1$	1.2378	.8519	.9777	1.2547	1.2339	-.0939
	(.0487)	(.0368)	(.0745)	(.0438)	(.0599)	(.0627)
$\hat{\alpha}_2$	-.0132	.1378	.0096	.0860	-.2869	-.1506
	(.0595)	(.0441)	(.0567)	(.0568)	(.0627)	(.0452)
$\hat{\alpha}_3$	-.2188	-.0005	.0053	-.3362	.0574	-.0512
	(.0309)	(.0331)	(.0462)	(.0301)	(.0385)	(.0415)
$\hat{\alpha}_4$	-.0068	.0099	.0035	-.0047	-.0051	-.1211
	(.0060)	(.0088)	(.0363)	(.0035)	(.0188)	(.0376)
$\hat{\delta}^2$	.0002	.0001	.0008	.0001	.0025	.1268

Numbers inside parentheses are standard errors. $\hat{\delta}^2$ is the error variance.

Table 7. **ML and NLS estimates for the ARFIMA(1,d,0) model for AR(4) residuals**

Variable	ML estimates $\hat{d}$	ML estimates $\hat{\alpha}_1$	ML estimates $\hat{\delta}^2$	NLS estimates $\hat{d}$	NLS estimates $\hat{\alpha}_1$	NLS estimates $\hat{\delta}^2$	Geweke Porter Hudak
ip	-.0621	.0528	.0031	-.0631	.0544	.0031	.202
	(.0429)	(.0551)		(.0429)	(.0552)		(.112)
bank	-.1001	.0654	.0926	-.1007	.0659	.0926	.238
	(.0365)	(.0498)		(.0365)	(.0499)		(.121)
tt	-.0086	.0273	5.1148	-.0093	.0284	5.1113	-.118
	(.0461)	(.0582)		(.0458)	(.0579)		(.152)
fx	.0393	-.0169	9.0370	.0593	-.0549	8.7906	.116
	(.0418)	(.0530)		(.0387)	(.0494)		(.108)
r	.0370	-.0468	.0753	.0376	-.0474	.0752	-.142
	(.0410)	(.0520)		(.0412)	(.0521)		(.138)
cpi	.0373	-.0461	.0002	.0378	-.0468	.0002	.386
	(.0362)	(.0484)		(.0364)	(.0485)		(.144)
wpi	.0239	-.0296	.0002	.0249	-.0318	.0002	.168
	(.0357)	(.0482)		(.0354)	(.0479)		(.162)
credit	.0420	-.0553	.0001	.0421	-.0554	.0001	.575
	(.0308)	(.0443)		(.0309)	(.0444)		(.142)
M1	.0745	-.0845	.0008	.0747	-.0848	.0008	.402
	(.0330)	(.0455)		(.0331)	(.0456)		(.153)
M2	.0676	-.0787	.0001	.0680	-.0791	.0002	.626
	(.0306)	(.0439)		(.0307)	(.0440)		(.142)
sx	-.0069	.0084	.0025	-.0071	.0086	.0025	-.342
	(.0454)	(.0565)		(.0457)	(.0568)		(.172)
st	-.2032	.1675	.1314	-.2054	.1726	.1311	-.167
	(.9450)	(.0581)		(.0438)	(.0573)		(.120)

Numbers inside parentheses are standard errors. Geweke-Porter-Hudak denotes the respective differencing parameter estimate.

Table 8. **The rescaled range V statistics and the estimates of the Hurst exponent (q=0)**

	V	H
ip	1.28	0.59
bank	1.36	0.49
tt	1.34	0.46
tx	1.66	0.51
r	1.73	0.60
cpi	1.69	0.80
wpi	1.95	0.83
credit	2.77	0.70
M1	2.27	0.72
M2	2.56	0.66
sx	0.83	0.34
st	1.14	0.44

The graphs of the R/S series are presented in Figure 8. On the basis of this figure, one may conclude whether the slope (i.e., the estimate of the Hurst exponent) is almost constant over all data points. The 5 per cent confidence interval for the V statistic is 0.85–1.75.

Table 9. **Estimation results of a nonlinear AR model**

	a_0	a_1	a_2	a_3	a_4	a_5	a_6	SEE	DW	F3
ip	.273	.105	.580	.201	.043	−.623	.223	.056	2.08	1.79
	(2.81)	(3.10)	(10.5)	(3.13)	(2.23)	(2.15)	(0.26)			
bank	.921	.071	.273	.146	.096	−.761	−.013	.318	2.24	17.80
	(3.77)	(1.35)	(3.41)	(1.24)	(2.88)	(2.45)	(0.34)			
tt	.216	.008	1.110	−.603	.333	−4.427	−5.617	.023	2.06	9.96
	(2.45)	(1.47)	(12.7)	(4.49)	(2.51)	(2.82)	(4.02)			
fx	.416	.002	.830	−.819	.648	−7.859	−1.701	.030	1.90	14.71
	(4.05)	(0.49)	(8.83)	(4.98)	(4.13)	(4.34)	(4.33)			
r	.005	.000	.774	.027	2.006	−59.76	181.12	.276	1.98	0.74
	(0.85)	(0.16)	(8.08)	(0.17)	(0.86)	(0.97)	(0.83)			
cpi	−.002	.033	1.344	−.340	−.002	.047	3.467	.013	2.14	10.65
	(1.44)	(3.31)	(31.9)	(8.03)	(3.31)	(2.42)	(1.34)			
wpi	−.164	.024	1.563	−.497	−.007	.019	−12.64	.013	2.17	12.70
	(3.70)	(2.58)	(39.2)	(11.5)	(3.37)	(3.31)	(3.39)			
credit	−.017	.019	1.486	−.480	−.001	.000	−106.8	.010	2.17	19.89
	(0.93)	(1.93)	(36.0)	(11.5)	(1.03)	(0.42)	(5.14)			
M1	−.390	.115	.809	.292	−.007	.008	5.225	.029	2.00	9.70
	(3.44)	(4.00)	(20.4)	(6.39)	(3.74)	(3.35)	(3.06)			
M2	−.118	.023	1.037	−.011	−.001	.001	−.511	.011	2.00	17.55
	(1.52)	(1.25)	(25.5)	(0.25)	(1.63)	(1.12)	(0.12)			
sx	.006	.140	1.256	−.281	.001	−.006	.240	.050	1.98	0.89
	(0.96)	(3.32)	(31.3)	(7.00)	(0.62)	(0.25)	(0.25)			
st	−.492	1.375	.736	.080	.021	−.129	−.011	.362	2.03	37.29
	(7.44)	(7.45)	(18.5)	(1.92)	(6.94)	(4.28)	(0.40)			

The estimating equation is of the form: $x_t = a_0 + a_1 t + a_2 x_{t-1} + a_3 x_{t-2} + a_4(x_{t-1}x_{t-2}) + a_5(x_{t-1}^3 x_{t-2}) + a_6(x_{t-1} - x_{t-2})^3 + \mu_t$, where μ is the random term. If we restrict $a_4 = a_5 = a_6 = 0$, we end up with a standard linear model. F3 represents an F test statistic for this restriction. The corresponding 5 % (1 %) critical value(s) is 2.64 (3.86). ip denotes (log) industrial production, bank (log) bankruptcies, tt the terms of trade, fx the real exchange rate index, r yield on long-term government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money, sx the (log) Unitas stock price index and st the turnover in stock exchange. The sample period is 1922M5–1996M9. Coefficient a_5 has been divided by 1000.

Table 10. **Some stability test results**

	Average lag length		Stability tests	
	I	II	Chow	Dummy test
ip	1.44	1.80	5.08	5.18
bank	2.12	2.12	0.68	0.30
tt	0.42	0.74	3.06	3.24
fx	0.88	0.89	3.05	3.03
r	0.79	1.01	1.32	1.55
cpi	0.30	0.83	8.14	10.10
wpi	0.33	0.46	3.64	3.77
credit	0.48	0.38	2.51	2.41
M1	0.68	1.52	8.92	11.75
sx	0.72	0.68	3.77	4.52
5 %	..	..	2.22	2.38
1 %	..	..	3.04	3.34

The average lag length is computed for the depression periods (I) and non-depression periods (II). Chow notes a Chow test statistic for the hypothesis that the coefficients of the AR(4) model are the same for these two subperiods. The dummy test denotes a F test for the multiplicative dummy*x_{t-1} -terms.

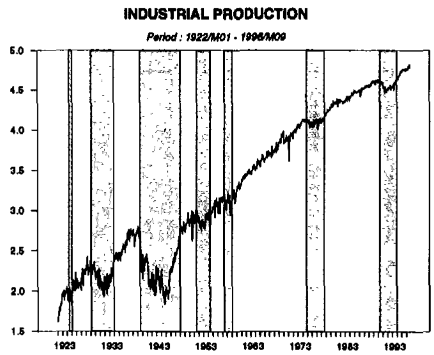
Table 11. **Number of significant Box–Ljung test statistics for the cross-correlation coefficients of different powers of AR(4) residuals**

	u	u^2	u^3	$\|u\|$	u_t
ip	3(6)	5(8)	2(6)	10(10)	1
bank	5(7)	8(9)	8(9)	10(10)	3
tt	5(1)	5(1)	5(1)	11(3)	1
fx	7(6)	5(7)	2(5)	8(9)	2
r	5(4)	5(6)	3(3)	8(9)	1
cpi	10(8)	7(6)	6(5)	9(8)	5
wpi	8(9)	8(7)	7(6)	9(10)	4
credit	10(8)	8(8)	8(7)	10(10)	5
M1	4(7)	3(3)	1(2)	6(8)	4
M2	7(6)	4(3)	1(0)	7(7)	2
sx	7(5)	7(7)	6(5)	10(11)	6
st	6(6)	7(7)	4(4)	8(11)	1

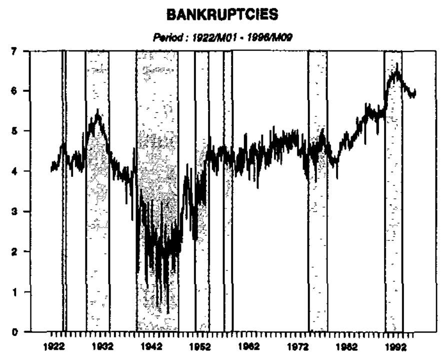
The first number indicates the number of significant Box–Ljung test statistics at the 5 per cent level of significance for the first 24 positive lags of the other variable. The second column (inside parentheses) indicates the corresponding number for the same number of leads. u indicates untransformed residuals, u^2 squared residuals, u^3 third power of residuals, $|u|$ absolute values of residuals and u_t contemporaneous values of residuals (these values are computed from simple correlation coefficients).

Figure 2. **Historical Finnish time series**

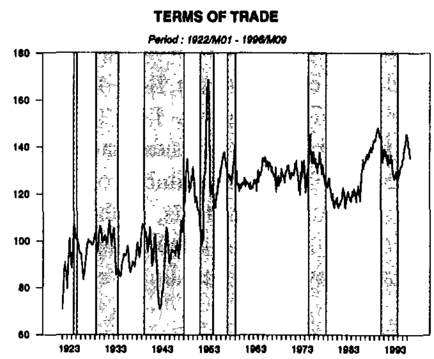
Industrial production



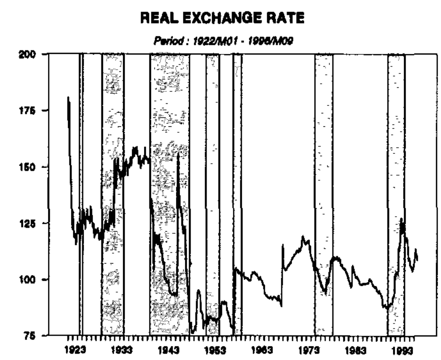
Bankruptcies



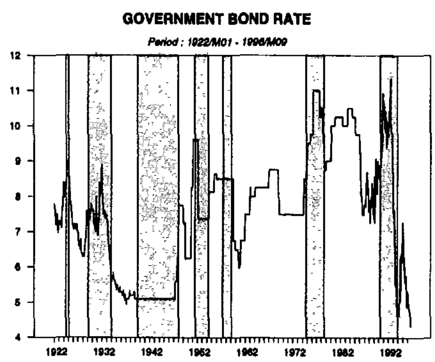
Terms of trade



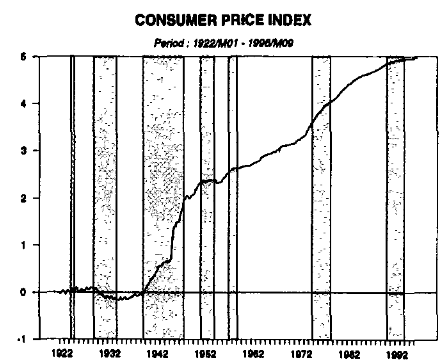
The real exchange rate index



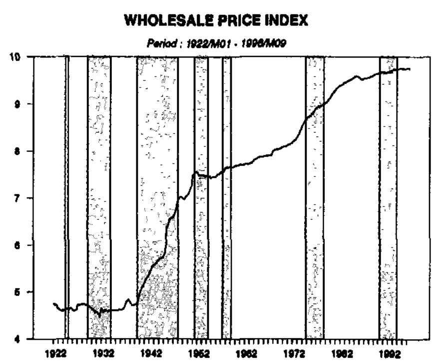
Yield on long-term government bonds



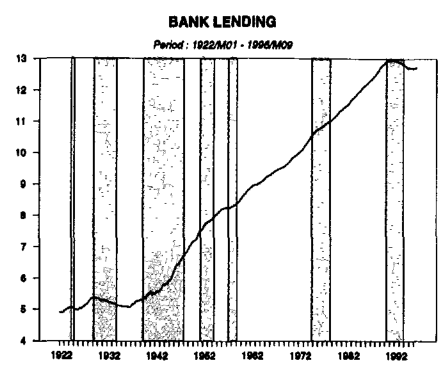
The consumer price index



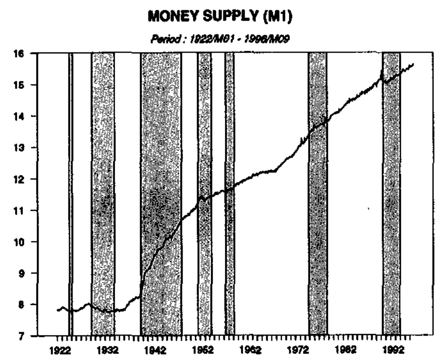
The wholesale price index



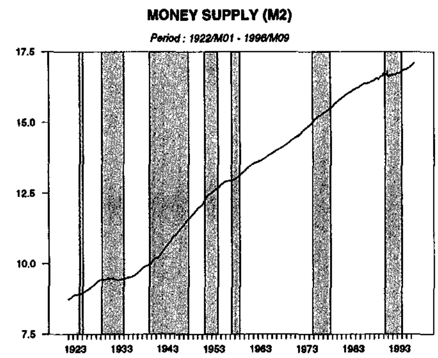
Bank's total credit supply



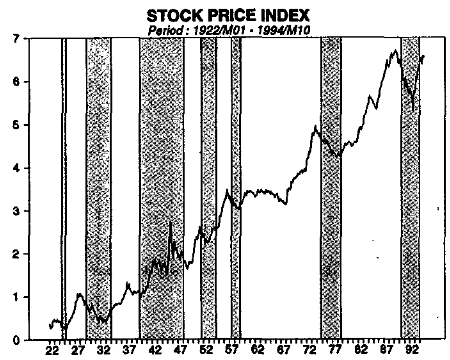
Narrow money (M1)



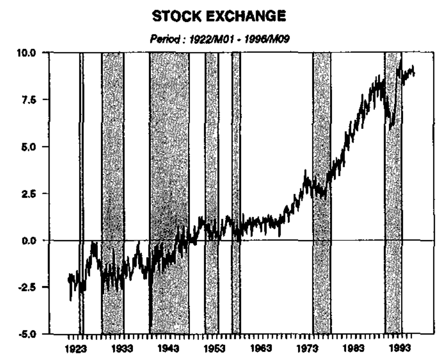
Money supply (M2)



The Unitas stock exchange index



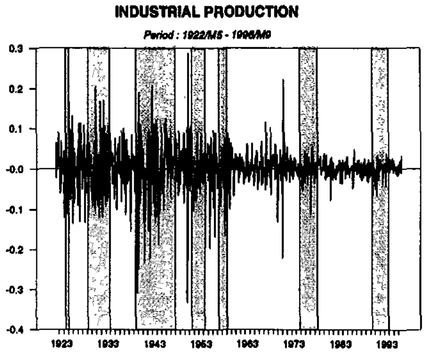
Turnover in Helsinki stock exchange



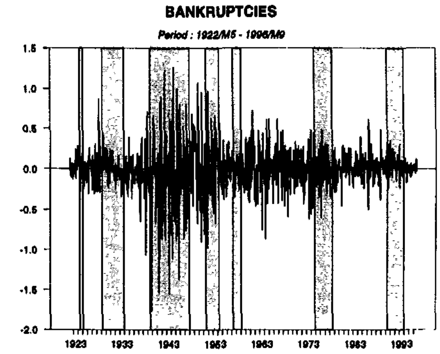
Nominal variables (except for the interest rate) are expressed in logs.

Figure 3. **Time series of AR(4) residuals**

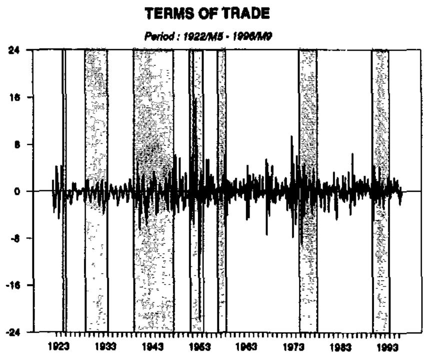
Industrial production



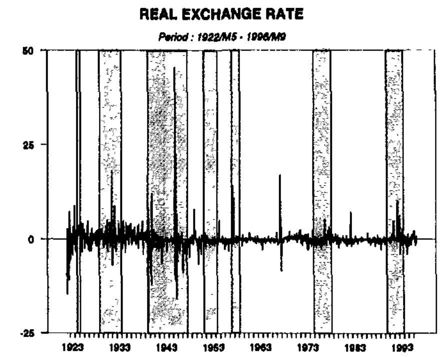
Bankruptcies



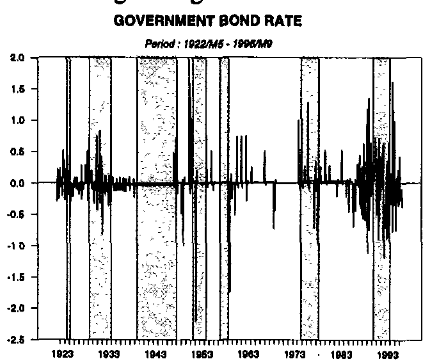
Terms of trade



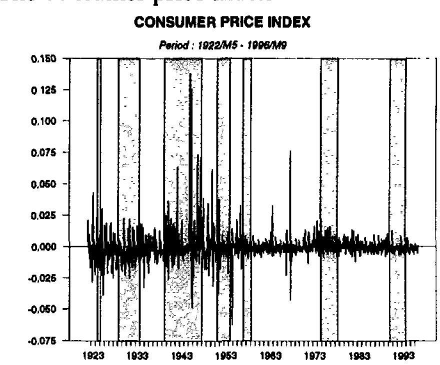
The real exchange rate index



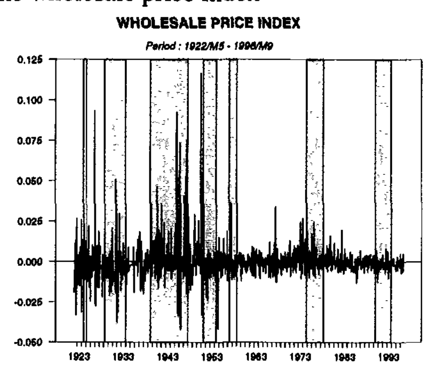
Yield on long-term government bonds



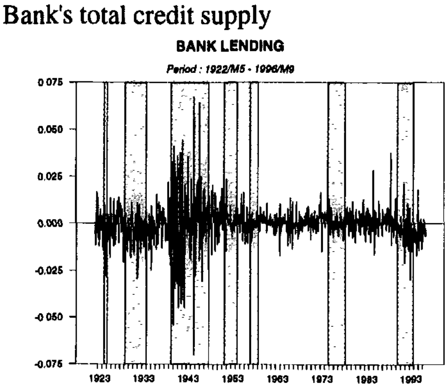
The consumer price index



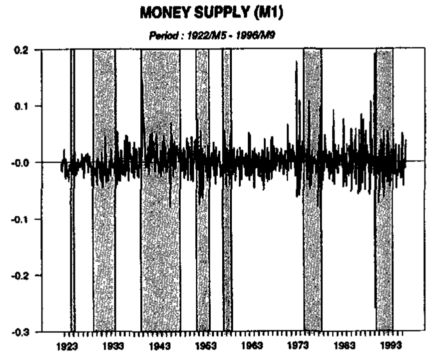
The wholesale price index



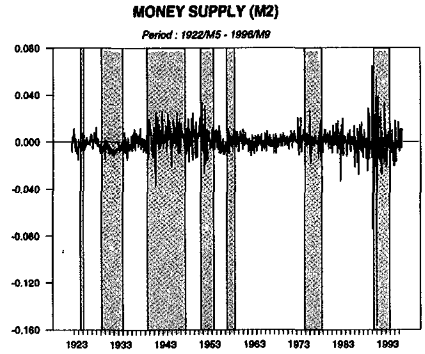
Bank's total credit supply



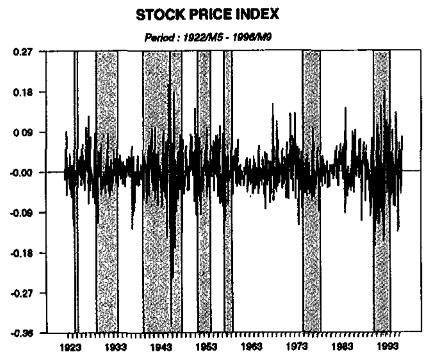
Narrow money (M1)



Broad money (M2)



The Unitas stock exchange index



Turnover in Helsinki stock exchange

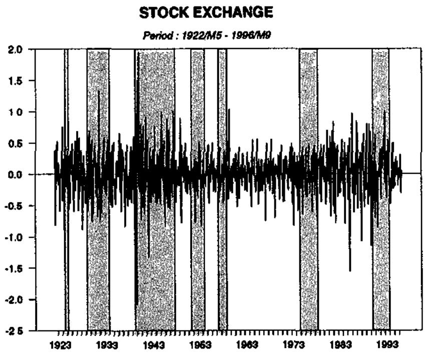
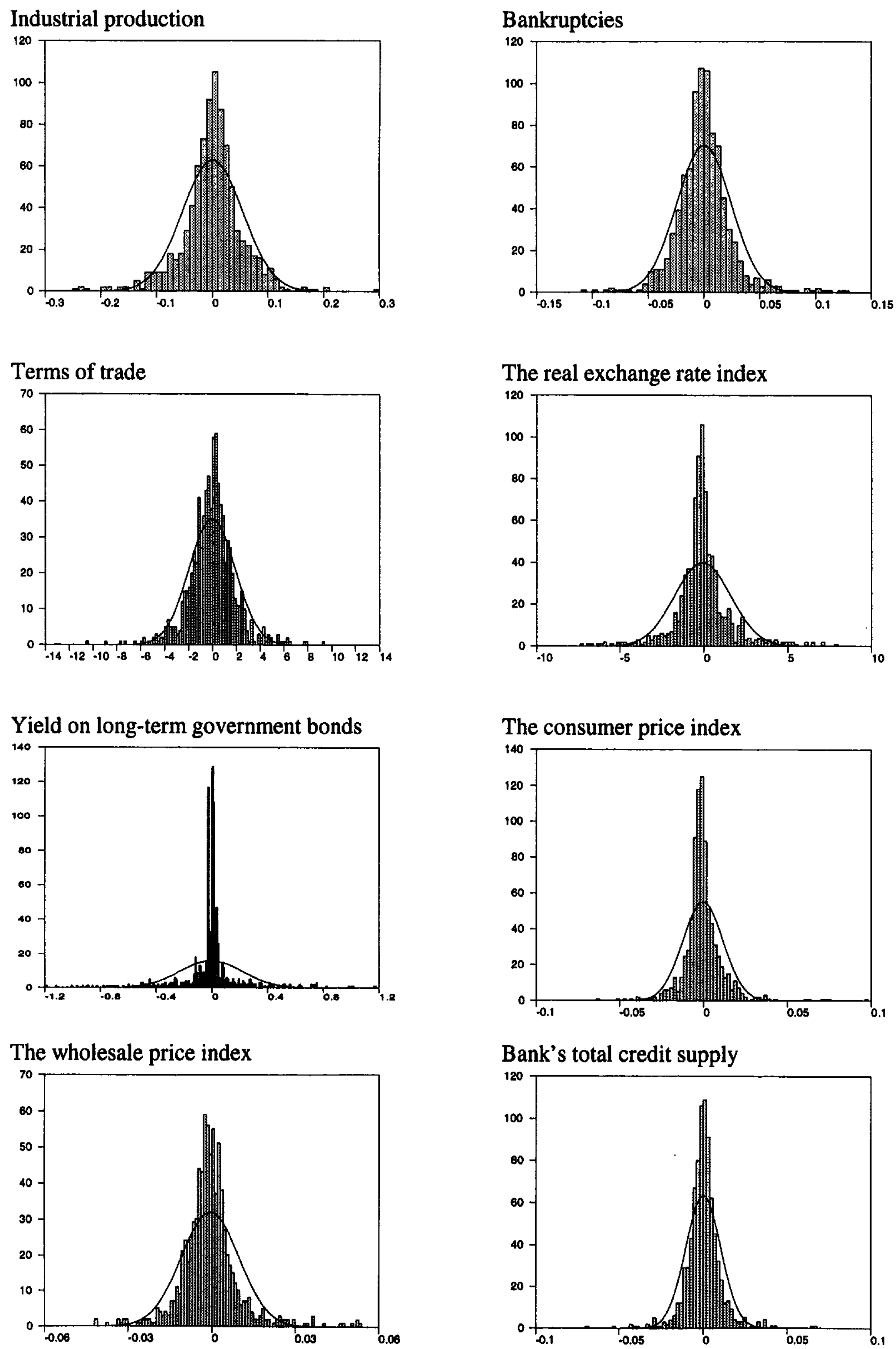
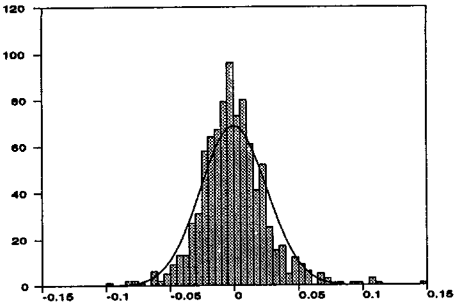


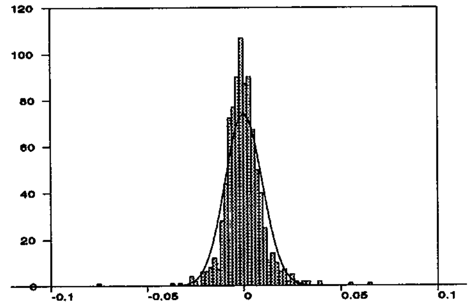
Figure 4. **Frequency distribution of AR(4) residuals**



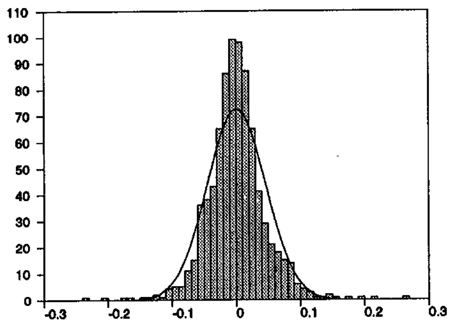
Narrow money (M1)



Money supply (M2)



The Unitas stock exchange index



Turnover in Helsinki stock exchange

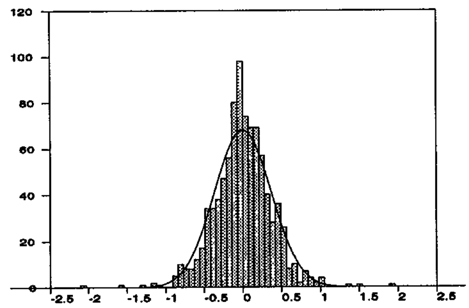
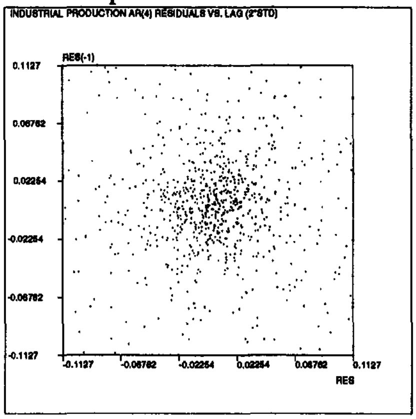
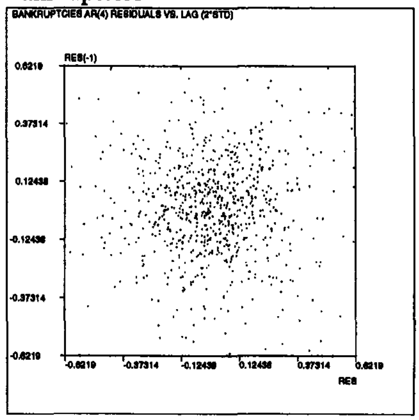


Figure 5. **Two-dimensional plots of AR(4) residuals**

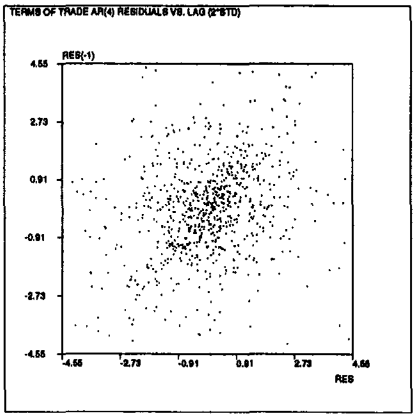
Industrial production



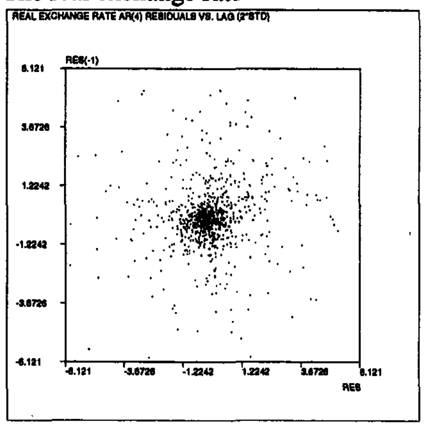
Bankruptcies



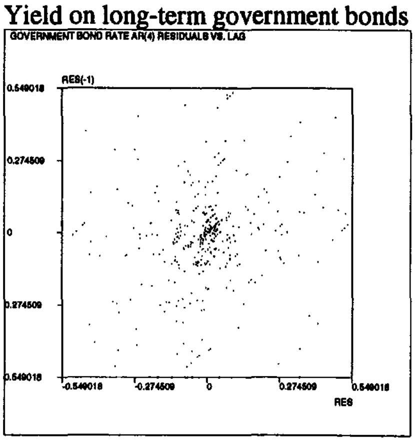
Terms of trade



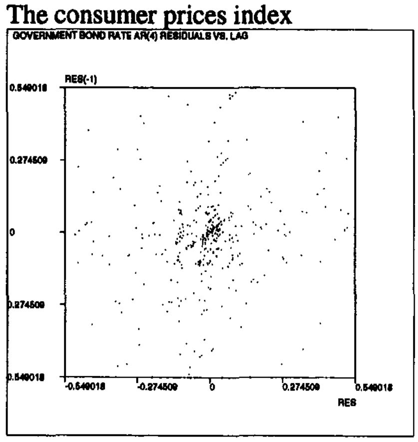
The real exchange rate



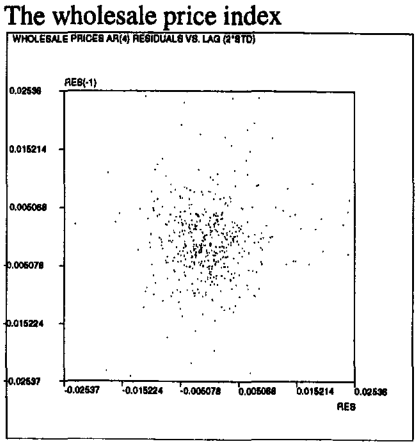
Yield on long-term government bonds



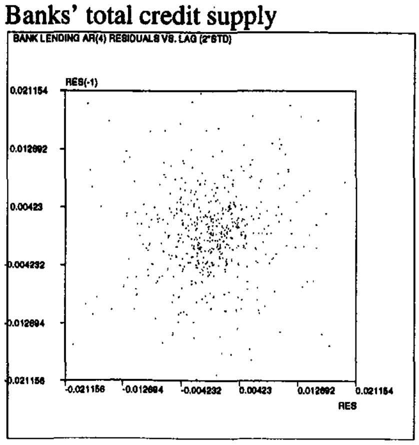
The consumer prices index



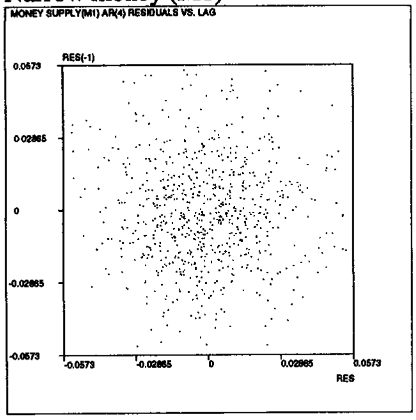
The wholesale price index



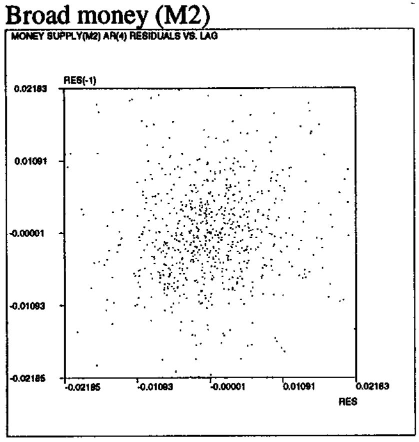
Banks' total credit supply



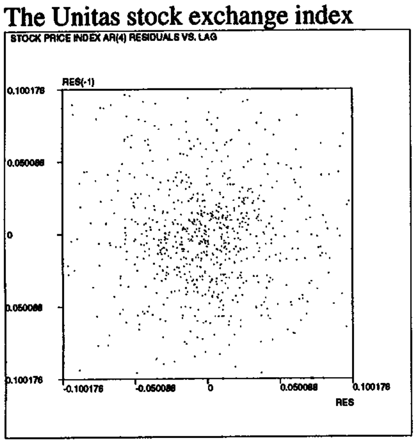
Narrow money (M1)



Broad money (M2)



The Unitas stock exchange index



Turnover in Helsinki stock exchange

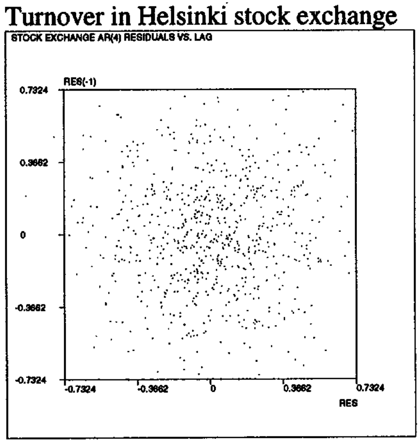
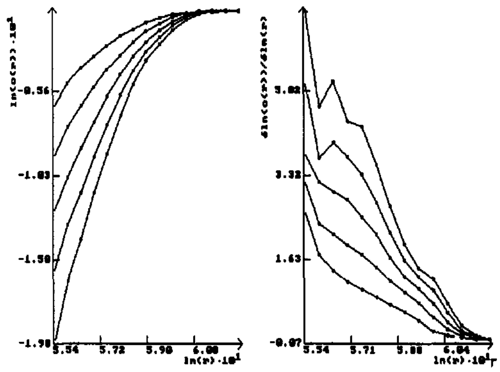
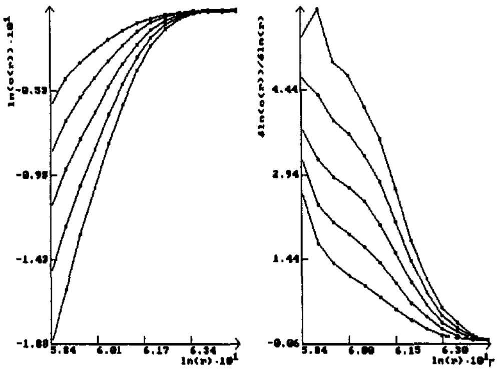


Figure 6. **Correlation dimension estimates**

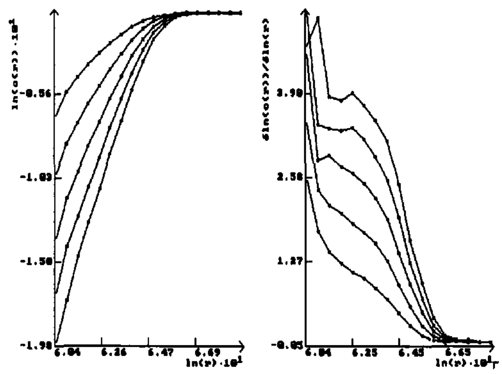
Industrial production



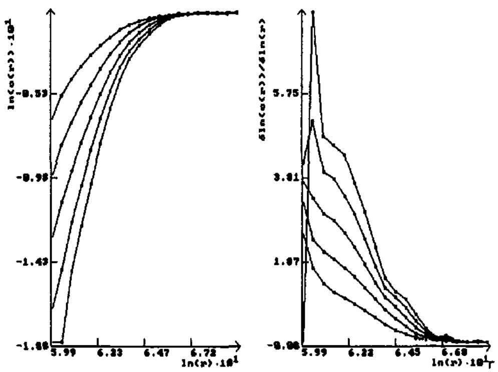
Bankruptcies



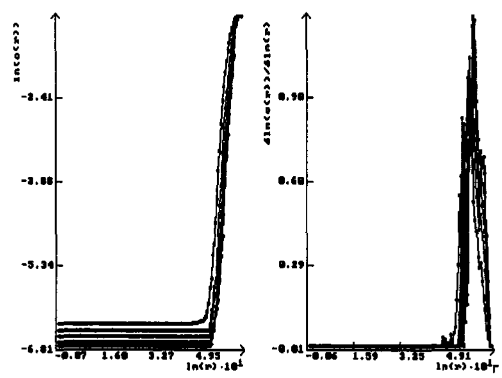
Terms of trade



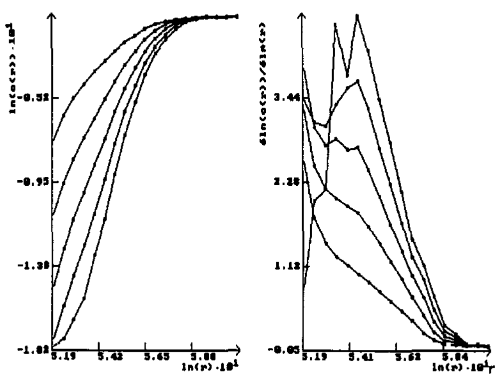
The real exchange rate



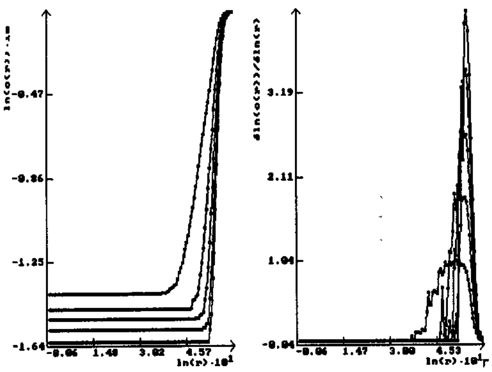
Yield on long-term government bonds



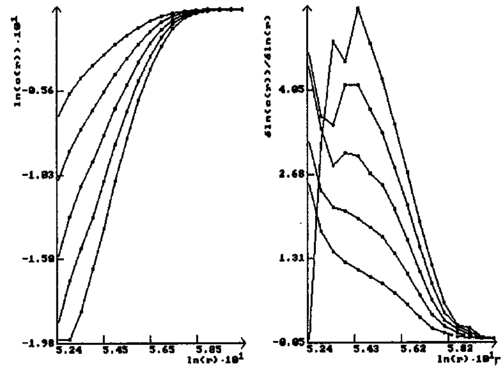
The consumer price index



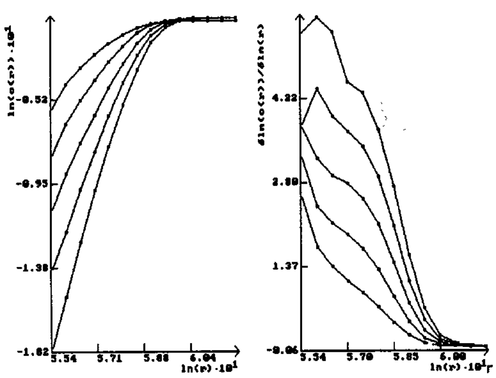
The wholesale price index



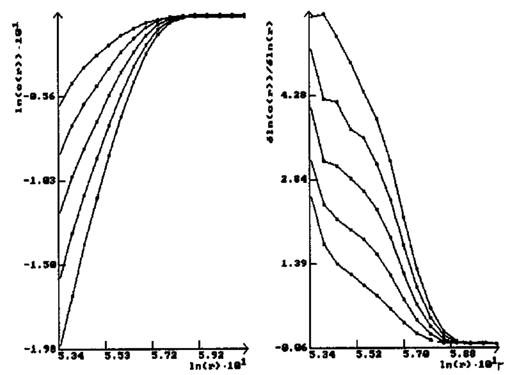
Bank's total credit supply



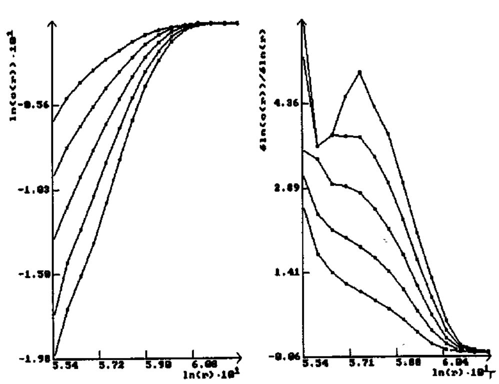
Narrow money (M1)



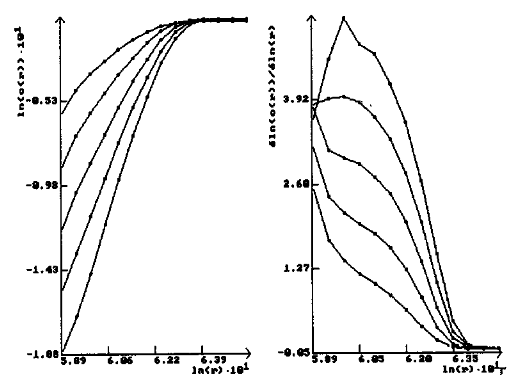
Money supply (M2)



The Unitas stock exchange index



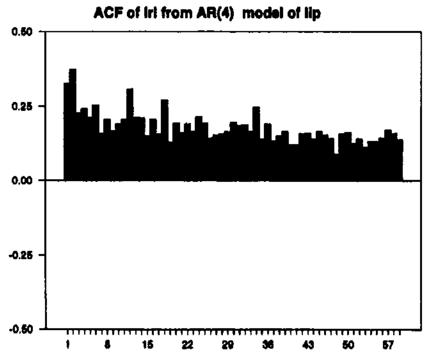
Turnover in Helsinki stock exchange



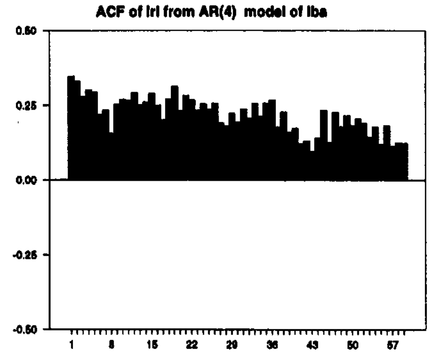
The x-axis in all figures is 10*log(ϵ), the y-axis in the left-hand- side figure is 10*log(C_m(ϵ)) and in the right-hand-side figure ∂log(C_m(ϵ))/∂log(ϵ).

Figure 7. **Autocorrelations of absolute values of AR(4) residuals**

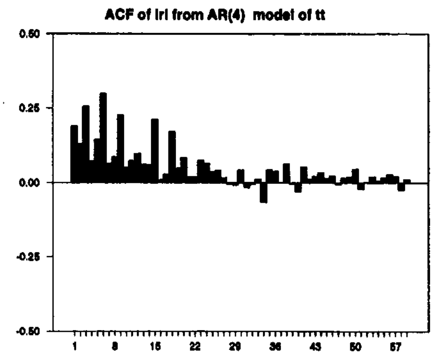
Industrial production



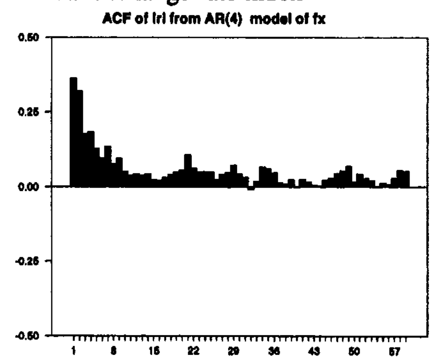
Bankruptcies



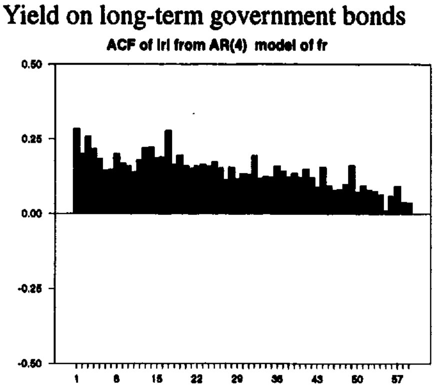
Terms of trade



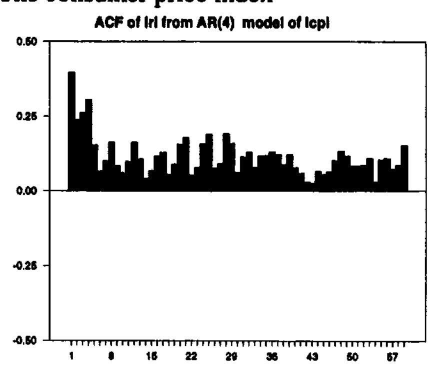
The real exchange rate index



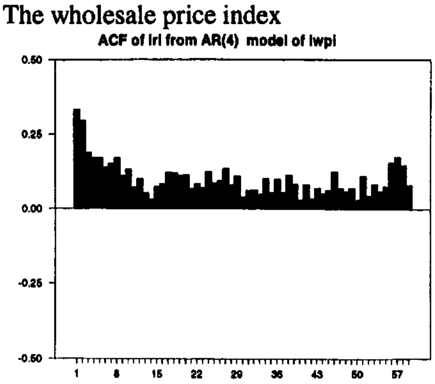
Yield on long-term government bonds



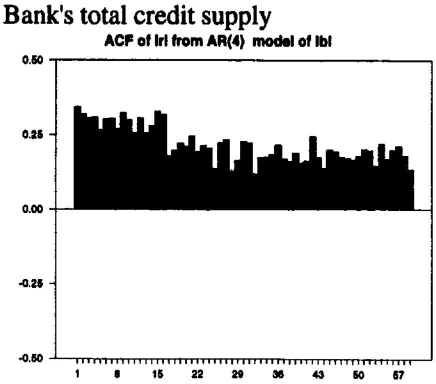
The consumer price index



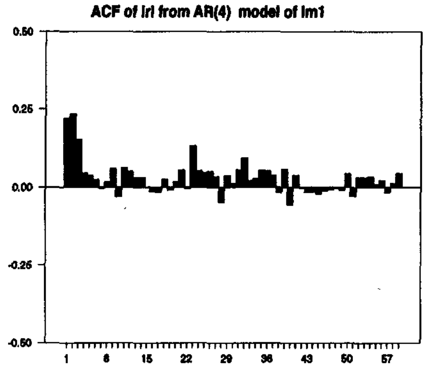
The wholesale price index



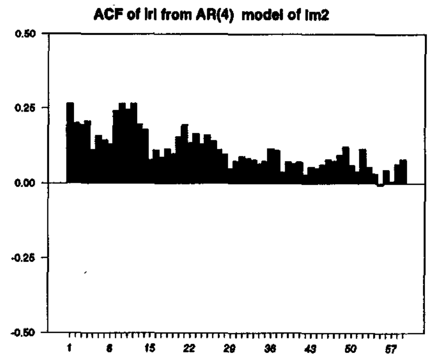
Bank's total credit supply



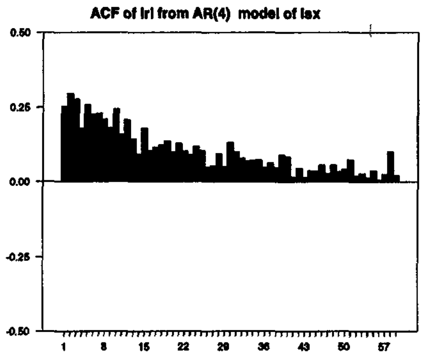
Narrow money (M1)



Money supply (M2)



The Unitas stock exchange index



Turnover in Helsinki stock exchange

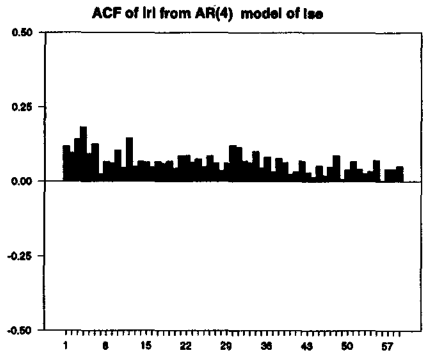
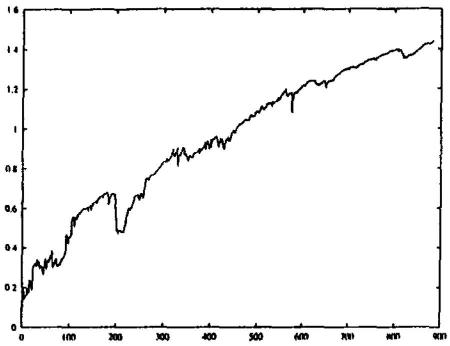
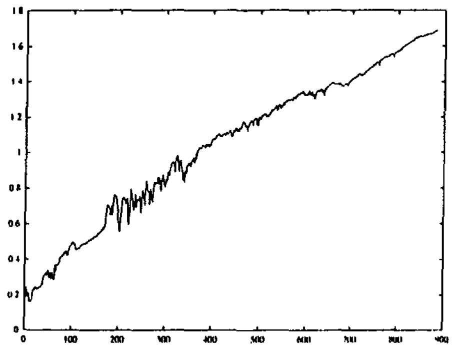


Figure 8. **Rescaled range estimates**

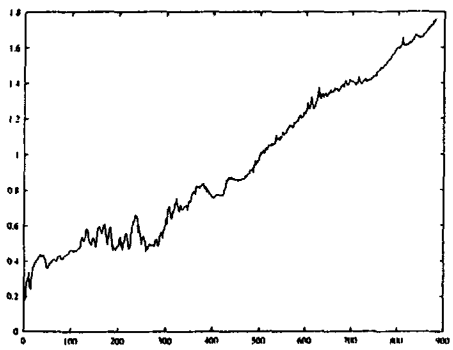
Industrial production



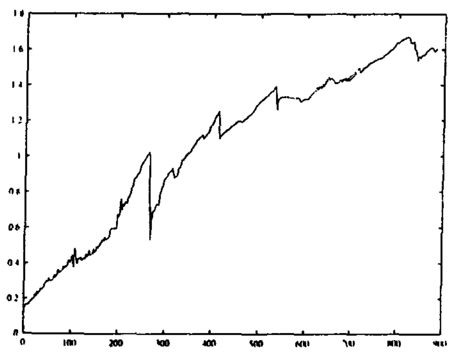
Bankruptcies



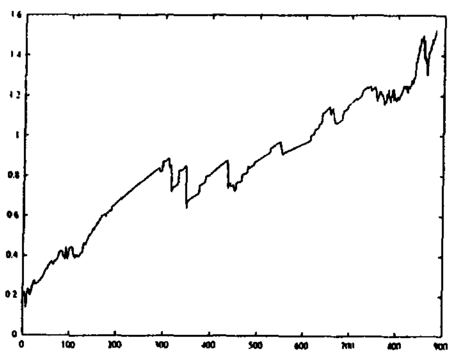
Terms of trade



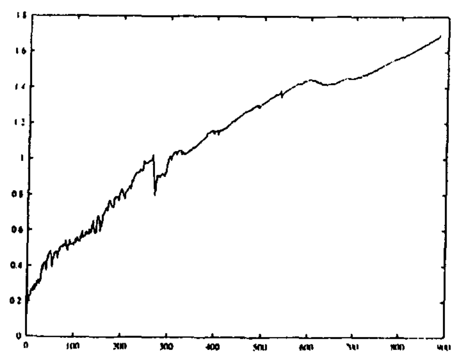
The real exchange rate index



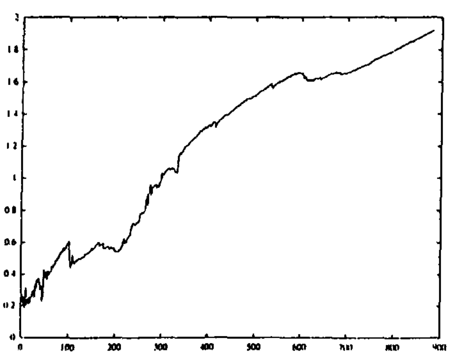
Yield on long-term government bonds



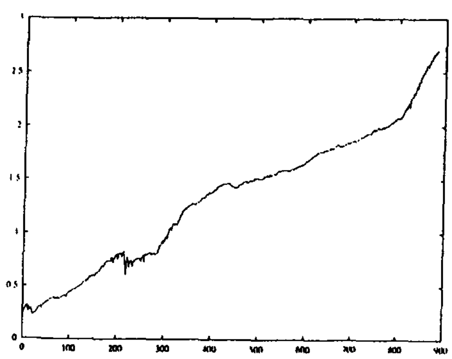
The consumer price index



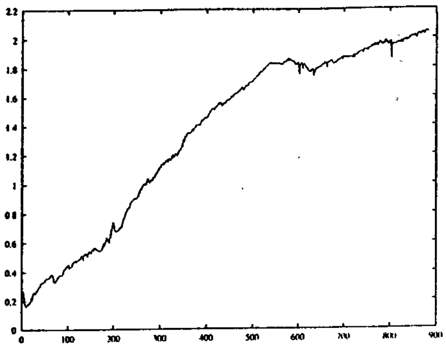
The wholesale price index



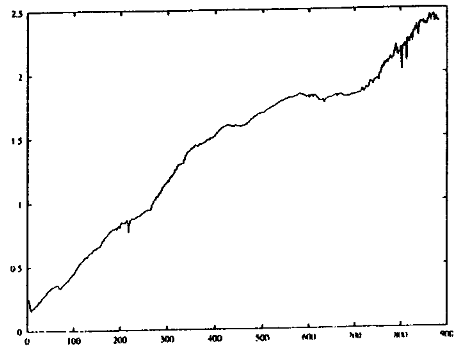
Bank's total credit supply



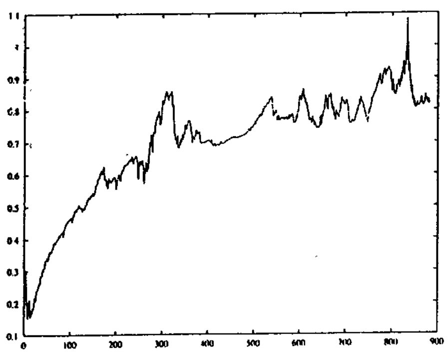
Narrow money (M1)



Broad money (M2)



The Unitas stock exchange index



Turnover in Helsinki stock exchange

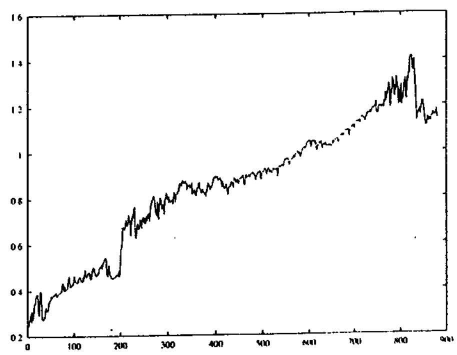
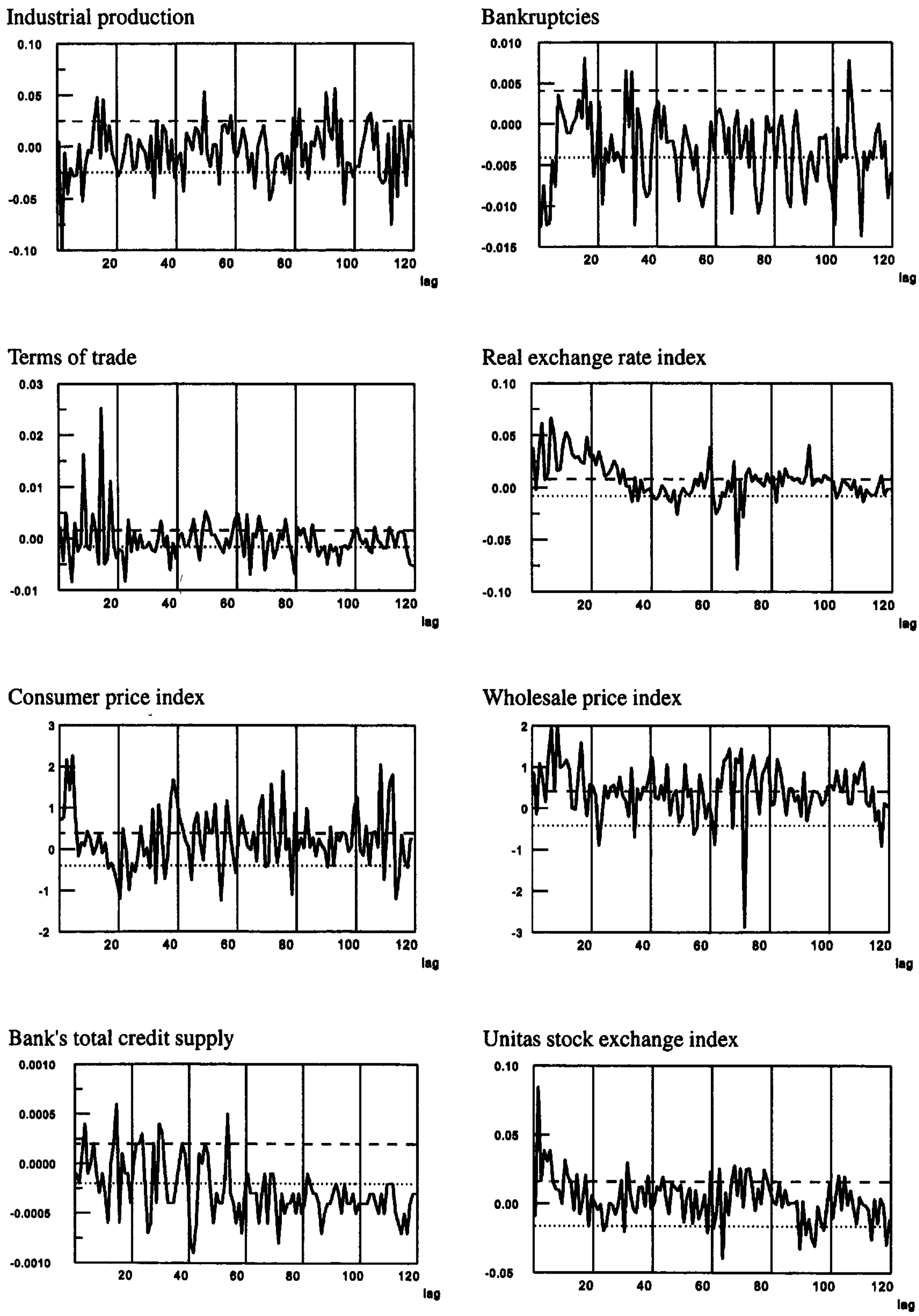


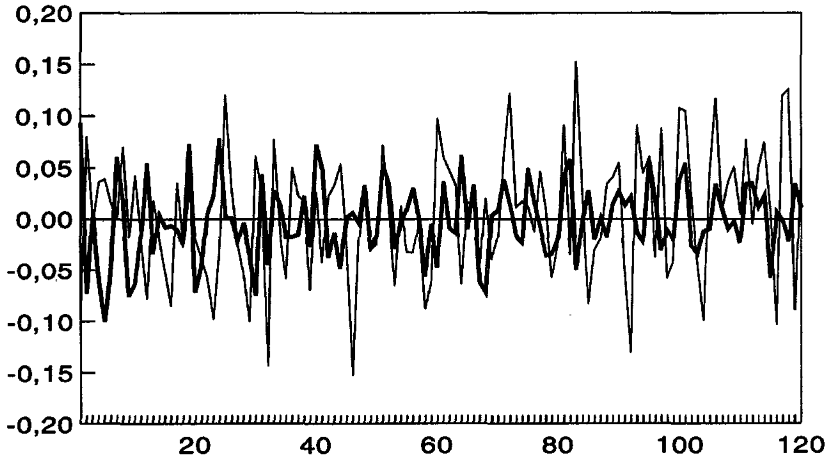
Figure 9.1 **Ramsey irreversibility test statistics**



The solid line denotes the $G^k_{2,1}$ test statistic; above and below it are the corresponding 5 per cent confidence limits.

Figure 9.2 **Ramsey irreversibility test statistics for ip and sx**

$G_{3,1}^{k}$-statistic for ip (thin line) and sx (bold line)



$G_{2,1}^{k}$-statistic for ip (thin line) and sx (bold line)

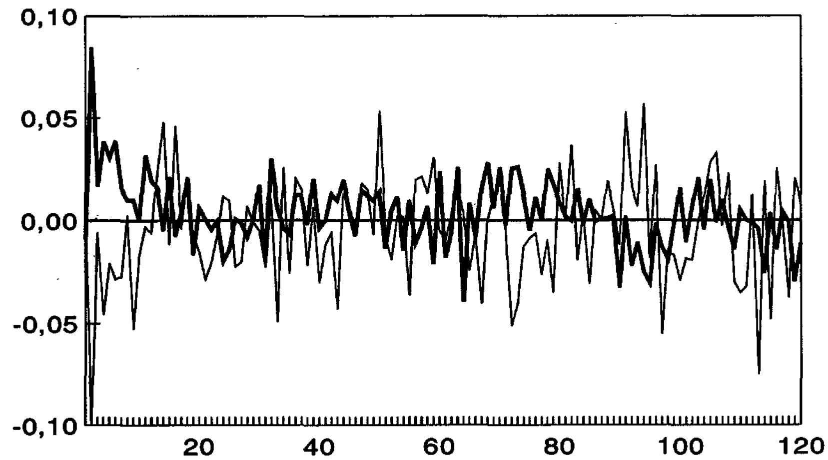
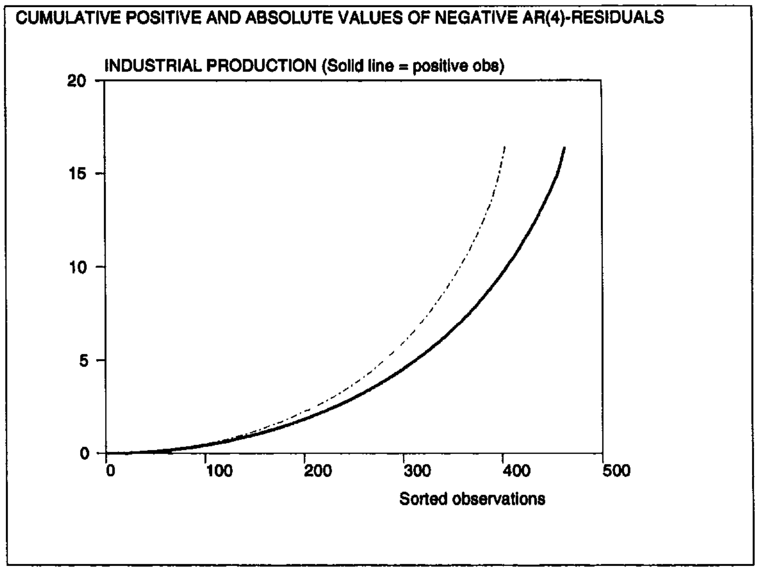


Figure 10. **Residuals for industrial production and stock prices**

Positive AR(4) residuals of industrial production ———
Absolute values of negative AR(4) residuals of industrial production -.-.-.



Positive AR(4) residuals of stock prices ———
Absolute values of negative AR(4) residuals of stock prices -.-.-.

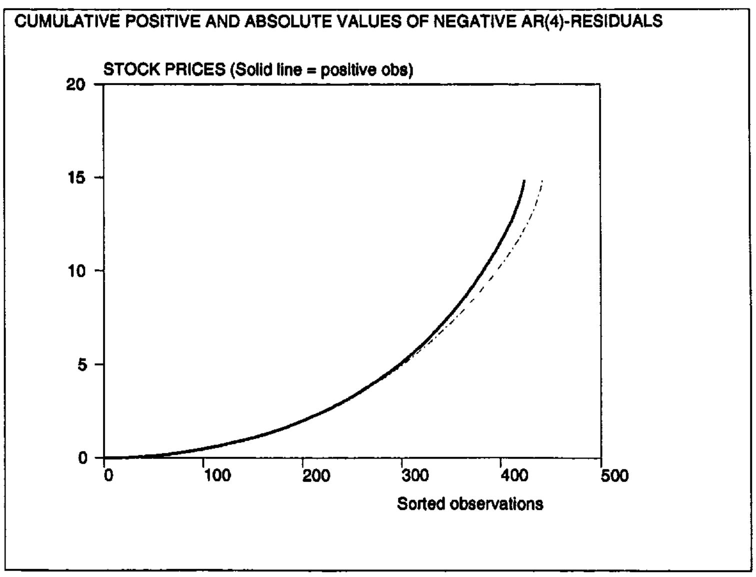
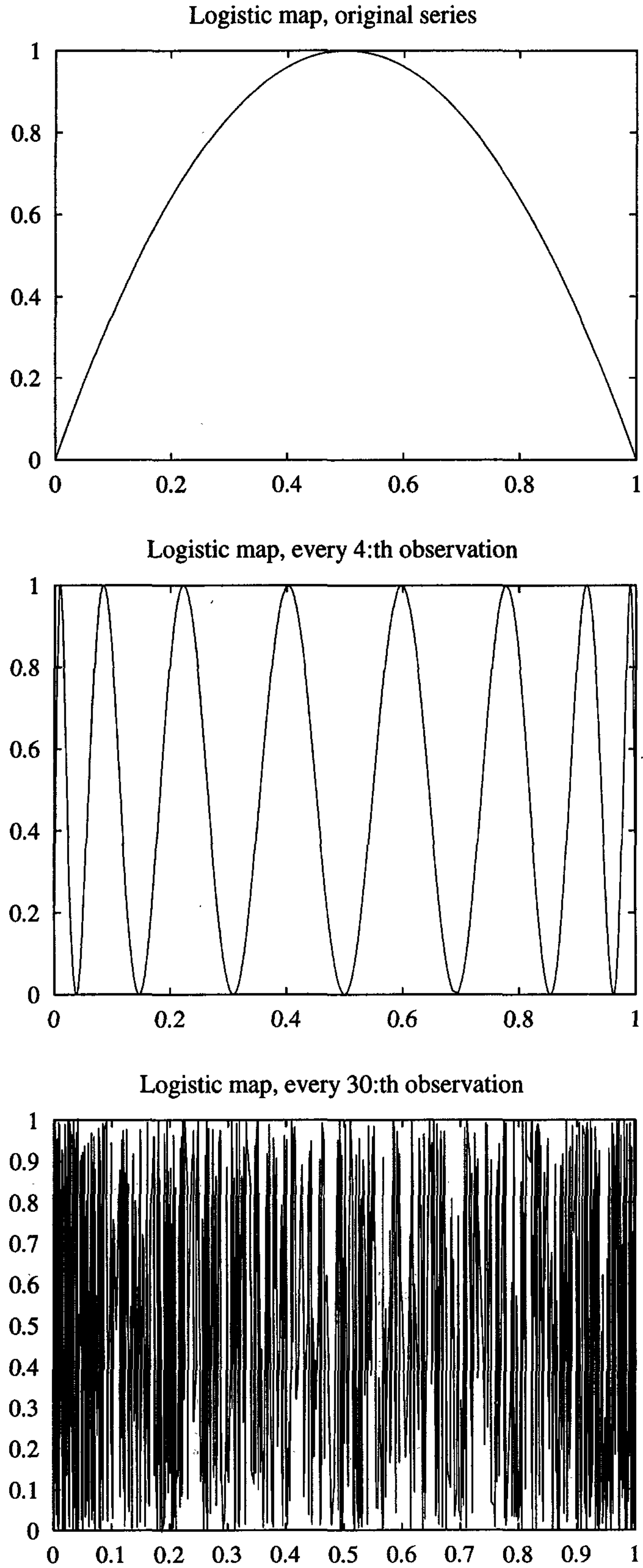


Figure 11. **Effect of sampling on two-dimensional plots**



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