DISSERTATIO MATHEMATICA.

DE

TRACTORIIS SECTIONUM CONICARUM

Cujus PARTEM PRIMAM

Conſ. Ampliſs. Facult. Philoſ. Aboënſ.

PRÆSIDE

Mag. ANDR. JOH. METHER

Math. Prof. Reg. & Ord.

publico examini ſubjicit

AXEL. JOSIAS FLODBERG

Stip. Brem. Aboenſis.

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ABOÆ, Typis FRENCKELLIANIS.

§. 1.

Quamvis doctrinam de lineis secundi ordinis sive de Sectionibus Conicis uberrime tractaverint & veteres & recentiores Mathematum Doctores, eo videlicet ex fundamento, quod tam sæpe in explicationibus phænomenorum naturalium occurrant, ut iis maxime quasi delectari videatur natura; minime tamen affectiones omnes atque proprietates harum linearum ita sunt determinatæ, ut tota hæc disciplina absoluta jam statui possit. Problemata etenim omnia a calculo integrali pendentia, nondum satis cognita dici posse vel ex eo intelligitur, quod ipse calculus ita sit comparatus, ut regulæ quæ hactenus inventæ sunt, generales nuncupari non possint. Ex his itaque videtur, doctrinam linearum secundi ordinis, elegantem omnino præbere materiam vires artesque Geometrarum exercendi, quapropter etiam, exercitium academicum edituri, naturam nobis proposuimus investigare lineæ curvæ, quam Tractoriæ nomine insignire solent Mathematici, & ejus est indolis, ut ducta e quovis puncto in curva data (quam quidem in sequentibus Sectionem quandam Conicam esse supponimus) linea quadam recta, hæc ipsa Curvam perpetuo tangat, juveniles nostros conatus censuræ B. L. jam submittentes.

§. 2.

Si fuerit ATL (fig. 1) curva quædam data, e qua ita ducta sit linea recta TM, ut aliam curvam Mm perpetuo tangat, curva Mm Tractoria dicitur ipsius ATL, & ad relationem inter coordinatas hujusce Tractoriæ inveniendam, sequens nobis commodissima videtur methodus. Sit videlicet C centrum circuli osculatorii curvæ ATL in puncto T, sumtoque puncto t infinite proximo ipsi T, ducantur radii curvaturæ CT & ct, atque demittatur tk ita ut ad angulos rectos insistat lineæ MT. Ductis deinde mt NT & nt, tangentibus curvarum Mm & ATL, sit q punctum occursus tangentium TM & tm, atque Q ipsorum NT & nt, jungantur puncta Q & q, eritque, posito $\angle NTM = v$, $dv = ntq - NTq = ntq - tQq + tQq - NTq = tqQ + (tQq - NTq =)\ QqT - NQt = tqT - NQt = tqT - tCT$; est enim $tqQ + QqT = tqT$ atque $tCTQ$ quadrilaterum, cujus anguli T & t sunt recti, adeoque $\angle tCT + \angle tQT = 180^\circ = \angle tQT + \angle NQt$, unde $\angle tCT = \angle NQt$. Positis $Sin\, v = \varphi$, $AT = s$, $Tt = ds$, $MT = t$, $CT = r$ & Sin tot. $= 1$, erit in $\Delta\, tkT$ rectangulo $1 : \varphi :: ds : tk = \varphi ds$, pariterque in $\Delta\, tkq$ rectangulo lo ob $> tqk$ infinite parvum, $t : \varphi ds :: 1 : > tqk = \frac{\varphi ds}{t}$, ac in $\triangle$ rectangulo tCT, existente $> tCT$ infinite parvo, $r : ds :: 1 : > tCT = \frac{ds}{r}$, unde $dv = > tqT - > tCT = \frac{\varphi ds}{t} - \frac{ds}{r}$. Erat autem $Sin\, v = \varphi$, adeoque $d\varphi = Cos\, v\, dv = dv \sqrt{1 - \varphi^2}$ ergo $dv = \frac{d\varphi}{\sqrt{1 - \varphi^2}}$. Comparatis vero jam hisce dv valoribus, habebitur $\frac{d\varphi}{\sqrt{1 - \varphi^2}} = \frac{\varphi ds}{t} - \frac{ds}{r}$. Est vero præterea in $\triangle$ tTk rectangulo $tT : Tk :: 1 : (cos\, v. =) \sqrt{1 - \varphi^2}$ unde $Tk = ds \sqrt{1 - \varphi^2}$ atque $Tk = TM + Mm - tm$, & hinc posito $Mm = dz$, $Tk = t + dz - (t + dt) = dz - dt$, adeoque $ds \sqrt{1 - \varphi^2} = dz - dt$. Ductis denique PT, pt, MR & mr normaliter in AR, atque ST parallela ipsi AR, erit positis $AP = \xi$, $Pp = d\xi$, $PT = u$, $AR = x$, $Rr = dx$, $RM = y$, $mo = dy$ & $MS = y - u$, ob $\triangle$ $MTS \backsim \triangle$ Mom, $MT : MS :: Mm : Mo$, seu $t : y - u :: dz : dy$, unde $t = \frac{(y - u)\, dz}{dy}$. His vero jam tribus æquationibus $\frac{d\varphi}{\sqrt{1 - \varphi^2}} = \frac{\varphi ds}{t} - \frac{ds}{r}$ (A), $ds \sqrt{1 - \varphi^2} = dz - dt$ (B), & $t = \frac{(y - u)\, dz}{dy}$ (C), cognita insuper æquatione Curvæ datæ, & determinata alterutra quantitatum φ vel t, dabitur relatio inter coordinatas Tractoriæ quæsitæ, quæ seorsim pro quavis sectione Conica nobis jam est investiganda.

§. 3.

Sit Linea ATP (Fig. 2) recta, atque $MT = t$ tangens Tractoriae, quem quidem constantem supponimus $= b$, & $\sin MTP = \varphi$, $PM = y$, $mn = dy$, $AP = x$ & $Pp = dx$, Comparatis jam equationibus (A) & (B) habebitur $ds = \frac{b\,d\varphi}{\varphi\sqrt{1-\varphi^2}} = \frac{dz}{\sqrt{1-\varphi^2}}$, seu $\frac{b\,d\varphi}{\varphi} = dz$, existente videlicet pro linea recta $r = \infty$, & assumto tangente t constante evanescunt quantitates $\frac{ds}{r}$ & dt. Quo autem determinetur quantitas φ, resumatur aequatio (C), quumque pro linea recta sit $u = o$, habebitur $b = y\frac{dz}{dy}$, unde $dz = \frac{bdy}{y}$ adeoque $\frac{bd\varphi}{\varphi} = \frac{bdy}{y}$, seu $\frac{d\varphi}{\varphi} = \frac{dy}{y}$, & peracta integratione $Log\,\varphi = Log\,y + Log\,C$, & transeundo a Logarithmis ad quantitates absolutas, $\varphi = Cy$. Jam vero quantitas ista corrigens est determinanda, eo ex fundamento, quod si fuerit $\varphi = 1$, erit $b = y$ adeoque habebitur $\varphi = \frac{y}{b}$. Invento autem valore ipsius φ, si substituatur in equatione $\frac{bd\varphi}{\varphi} = dz$, eruitur $\frac{b\,dy}{y} = dz = \sqrt{dx^2 + dy^2}$, unde denuo $dx = \frac{dy\sqrt{b^2 - y^2}}{y}$. Hujus autem equationis integrale, quo facilius innotescat, statuatur $\sqrt{b^2-y^2} = p$, ideoque $\frac{dy\sqrt{b^2-y^2}}{y} = -\frac{p^2 dp}{b^2-p^2}$; est vero $\int\frac{-p^2 dp}{b^2-p^2}$ $= \int dp - \frac{b^2 dp}{b^2-p^2} = p + \frac{1}{2} b \text{ Log. } \frac{b-p}{b+p}$, ergo $x = p$ $+ \frac{1}{2} b \text{ Log. } \frac{b-p}{b+p} = \sqrt{b^2-y^2} + \frac{1}{2} b \text{ Log. } \frac{b - \sqrt{b^2-y^2}}{b + \sqrt{b^2-y^2}}$.

Hæc equatio jam exhibet relationem inter coordinatas Tractoriæ, quæ hoc in casu, quo linea *AT* recta est, simplex vocari solet.

Quod ad rectificationem curvæ jam inventæ attinet, facillime ope formulæ generalis $\sqrt{dx^2+dy^2}$ investigari potest. Habuimus enim supra $dx =$ $= \frac{dy\sqrt{b^2-y^2}}{y}$, unde $\sqrt{dx^2+dy^2} = \frac{b dy}{y}$; atque $\int\sqrt{dx^2+dy^2}$ $= b \text{ Log } y + C$

Quadraturam Curvæ nostræ ex quadratura Circuli pendere, cuique apparet. Erat enim $dx = \frac{dy\sqrt{b^2-y^2}}{y}$, adeoque elementum areæ $ydx = dy\sqrt{b^2-y^2}$, quæ quidem expressio, in seriem infinitam resoluta dabit $bdy - \frac{y^2 dy}{2b} - \frac{y^4 dy}{8b^3} - \frac{y^6 dy}{16 b^5} - \frac{5 y^8 dy}{118 b^7} -$ &c. adeoque $\int ydx = by - \frac{y^3}{6b} - \frac{y^5}{40b^3} - \frac{y^7}{112 b^5} - \frac{5y^9}{1152 b^7} -$ &c. quam formulam esse areæ quadrantis Circuli radio b descripti notissimum est.

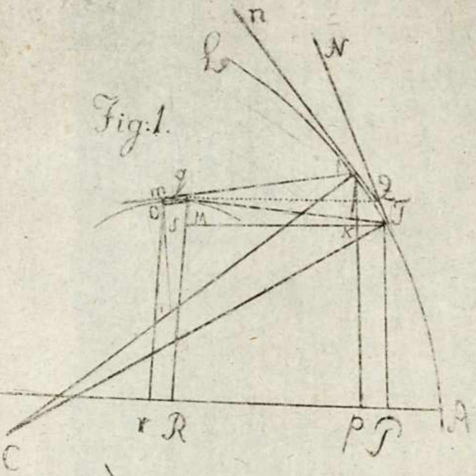

Fig:1.

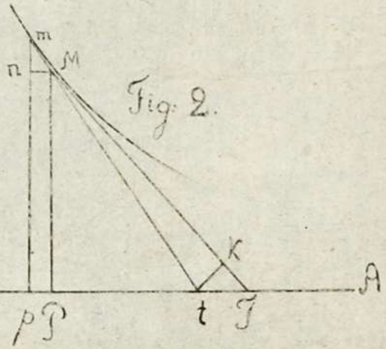

Fig: 2.

C:O. Sylv: fc.

Si vero desideretur Solidum ex revolutione Tractoriae simplicis circa axem genitum, absque prolixo calculo illud determinari potest, posita ratione Diametri ad peripheriam ut I: π. Nam formula generalis pro inveniendis solidis ex revolutione circa axem genitis $= \int 2\pi\, y^2\, dx$, redigitur facta debita substitutione in hanc formam $\int 2\pi y dy \sqrt{b^2 - y^2}$, cujus integrale $C - \frac{2\pi\,(b^2 - y^2)^{\frac{3}{2}}}{3}$ ipsum exhibet solidum. Pariter hujus Solidi superficies retenta eadem proportione inter Diametrum Circuli & ejus peripheriam ope formulae generalis $\int 2\pi y \sqrt{dx^2 + dy^2}$ eruitur, erit enim $\int 2\pi y \sqrt{dx^2 + dy^2} = \int 2\pi b dy = \pi b y. + C.$

Quod si vero tangens quem in solutione Problematis nostri constantem supposuimus fuerit variabilis, infinitas utique Tractoriarum species, pro diversis ipsius Tangentis valoribus oriri, neminem fugit. Nostras autem jam allatas aequationes (A) (B) & (C) ad aequationem Tractoriae inveniendam sufficere, perspicuum est; sed ipsum calculum, quo nimiam evitemus prolixitatem, omittimus.

Posito angulo MTP constante, nulla quidem datur Tractoria; aequatio etenim (A) $\frac{d\varphi}{\sqrt{1 - \varphi^2}} = \frac{\varphi ds}{t}$ dabit posito φ constante, $\frac{\varphi ds}{t} = o$, adeoque Tractoria secundum methodum quam supra adhibuimus, determinari nequit.

Schol.